Error Detection and Error Correction for PMU Data as Applied to Power System State Estimators

by

Veerakumar Murugesan

A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Science

Approved November 2013 by the
Graduate Supervisory Committee:

Vijay Vittal, Co-Chair
Gerald Heydt, Co-Chair
Raja Ayyanar

ARIZONA STATE UNIVERSITY

December 2013

ABSTRACT

In modern electric power systems, energy management systems (EMSs) are responsible for monitoring and controlling the generation system and transmission networks. State estimation (SE) is a critical 'must run successful' component within the EMS software. This is dictated by the high reliability requirements and need to represent the closest real time model for market operations and other critical analysis functions in the EMS. Traditionally, SE is run with data obtained only from supervisory control and data acquisition (SCADA) devices and systems. However, more emphasis on improving the performance of SE drives the inclusion of phasor measurement units (PMUs) into SE input data.

PMU measurements are claimed to be more accurate than conventional measurements and PMUs 'time stamp' measurements accurately. These widely distributed devices measure the voltage *phasors* directly. That is, phase information for measured voltages and currents are available. PMUs provide data time stamps to synchronize measurements. Considering the relatively small number of PMUs installed in contemporary power systems in North America, performing SE with only phasor measurements is not feasible. Thus a hybrid SE, including both SCADA and PMU measurements, is the reality for contemporary power system SE. The hybrid approach is the focus of a number of research papers.

There are many practical challenges in incorporating PMUs into SE input data. The higher reporting rates of PMUs as compared with SCADA measurements is one of the salient problems. The disparity of reporting rates raises a question whether *buffering* the phasor measurements helps to give better estimates of the states.

The research presented in this thesis addresses the design of data buffers for PMU data as used in SE applications in electric power systems. The system theoretic analysis is

illustrated using an operating electric power system in the southwest part of the USA. Various instances of state estimation data have been used for analysis purposes. The details of the research, results obtained and conclusions drawn are presented in this document.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my research advisors, Dr. Gerald Heydt and Dr. Vijay Vittal for their advice, guidance, and motivation in all stages of this project. Their expertise, insight and useful feedback have been crucial in performing this research work. I am grateful to Dr. Raja Ayyanar for being a part of my graduate supervisory committee.

I would also like to extend my thanks to the Salt River Project (SRP) for providing me an opportunity to perform this research work. My specific thanks go to Dr. Naim Logic and Mr. Steve Sturgill for their valuable suggestions and comments. I would like to thank Salt River Project for providing useful data during different stages of this project.

I would like to thank Dr. Yacine Chakhchoukh for his invaluable support, suggestions and valuable comments for this project.

I am deeply indebted to my parents, Mr. G. Murugesan and Mrs. K. Shunmuga-thai, for their love, cooperation, help and support.

Furthermore, I would like to thank all my friends in Arizona State University for their constant support and help whenever I needed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure Page

NOMENCLATURE

AC	Alternating current
A/D	Analog to digital
buf_size	Size of the buffer during process
B_s	Shunt reactive power
e	Measurement error
EMS	Energy management systems
FACTS	Flexible AC transmission systems
GPS	Global positioning system
G_s	Shunt active power
$G(x)$	Gain matrix
$h(x)$	Measurement function
h_i	Hypothesis testing
H	Measurement jacobian
H_{lim}	Upper limit for mean shift
i.i.d.	Independent and identical distribution
init_std	Initial standard deviation
I	Current
$J(x)$	Objective function for SE
k	Variable for iteration index
L_{lim}	Lower limit for mean shift
limit_array	Array to hold variance thresholds
M_{buff}	Mean of present buffer

M_{prev}	Mean of previous buffer
MSD	Measurement standard deviation
n	Variable to refer size of elements
N	Largest size of the buffer
p	Variable to indicate norm type
P	Active power
PF	Power flow
P_G	Generator active power
P_L	Load active power
P_{max}	Generator maximum active power limit
P_{min}	Generator minimum active power limit
PMU	Phasor measurement unit
PPS	Pulse per second
Q	Reactive power
Q_G	Generator reactive power
Q_L	Load reactive power
Q_{max}	Generator maximum reactive power limit
Q_{min}	Generator minimum reactive power limit
r_{br}	Resistance
R	Covariance matrix
R	Software product for statistics
S	Fixed shunt
SE	State Estimation

SCADA	Supervisory control and data acquisition
sigma	Standard deviation
std. dev.	Standard deviation
std_thr	Variable for holding std. dev. threshold
TP	Topology processor
upperthreshold	Threshold value for variance
V	Voltage
V_a	Phasor voltage angle
V_m	Phasor voltage magnitude
V_n	Sample variance
$V_{scheduled}$	Generator scheduled voltage
WLS	Weighted least squares
x	State variable
X_{line}	reactance
X	Random sample
$\overline{X}$	Mean of random sample
x_i	Variable specified in norm calculation
$\mid X \mid_p$	p - norm
$\mid X \mid_\infty$	Infinite norm
z	Measurements
Δx	Change in state variable
σ	Measurement standard deviation

θ	Voltage phase angle
Γ	Gamma function

Chapter 1. A foreword on state estimation and phasor measurements

1.1 Background

Electric power systems have become increasingly large and complex over the course of time. Monitoring and controlling the power system has become more challenging and needs modernized energy management systems (EMSs) [1]. EMSs require system measurements to perform their tasks. Input measurements are nominally attained using direct measurement of voltage and current magnitudes and active and reactive power measurements in AC systems. Because the accuracy of these measurements is limited, and because the data are time variable, and because conventional AC measurements do not give phasor information, a mathematical estimation technique has been applied to enhance the EMS measurement data. State estimation (SE), one of the key functions of an EMS, has gained importance. This is due to SE being crucial for system security analysis and influencing electric market decisions [2]. Traditionally, SE is conducted with data only from supervisory control and data acquisition (SCADA) units and systems. The drawbacks of the SCADA measurements are the inaccuracy due to communication latency, the time skew and absence of phase angle data [3].

Phasor measurement units (PMUs) can be used to compensate the problems in SCADA. This is because voltage phasor measurements are provided by PMUs and the device uses synchronization signals from the global positioning system (GPS) satellites to provide the positive sequence phasor voltages and currents at its location with a time stamp. However, all of the substations in a system do not have PMUs installed and hence it is not possible for PMUs to completely replace SCADA devices in the near future [4], [5]. There are several research efforts that claims better estimates in SE results with the inclusion of

PMUs, however, this claim depends on a number of factors such as measurement accuracy, number of PMUs installed and optimal PMU locations, related SCADA accuracy and calibration required. There are certain practical issues to be addressed before using PMUs along with SCADA such as synchronization of PMU data with one common reference, correction required between PMUs from different vendors and utilities, finding appropriate weights for PMU measurements relative to SCADA measurements and test studies to show actual improvement in estimates after including PMUs. Active research is currently being done to demonstrate improved state estimates with both SCADA data and synchronized phasor measurements from PMUs [6].

1.2 Power system state estimation

State estimation depends largely on statistical characteristics of the measurements as well as certainty of the network model and hence SE is not deterministic [7]. SE can be treated as a transformation between the input measurements from SCADA and the output states. The input for the SE is raw measurements from the field that are received at the control center and estimation is performed using these measurements. Voltage magnitudes and relative phase angles of all buses in the system is a part of the output from SE. These outputs are collectively called as voltage phasor which is one of the most important system states. The phasor voltage estimates (magnitude and phase) are used to calculate the real time active and reactive power flows. This provides a clearer picture of operating conditions than the measurements which may contain gross errors. Necessary measures in order to keep the system secure will be taken based on these operating conditions [8]. A state estimator typically includes the following modules:

Topology processor (TP): A TP converts the detailed breaker / switch model of network into a bus / branch model required for SE [9]. The accuracy of the network model developed for representing the connected system impacts the performance of the SE. Hence, updating the network configuration to represent the actual system in the field is necessary.

Observability analysis: Observability analysis follows topology processing and is needed to ensure if the set of available measurements is enough to find the unique estimate of the system states. In case, only a part of the system is observable, only that observable island could be estimated [10]. Pseudo-measurements could be added to restore observability to the unobservable part of the system.

Bad data detection and identification: Identifying the erroneous measurements and minimizing their effect are essential for solving the SE problem effectively. This is done by either removing bad measurements or assigning less weights to them. Otherwise, the SE results would get distorted [11]. That is, if the weights are not selected accurately, the SE solution will not be the maximum likelihood solution. Most of the existing SEs currently perform bad data detection and identification as a post processing step after actual SE is done [7].

1.3 Implementation of state estimation

State estimation in AC systems is a nonlinear problem and requires an iterative solution. SE is usually formulated as a weighted least squares (WLS) algorithm which involves minimizing the following objective function [12],

$$J(x) = \sum_{i}^{m} \frac{(z - h(x))^2}{R_{ii}} \tag{1.1}$$

$$= [z - h(x)]^T R^{-1} [z - h(x)] \tag{1.2}$$

In the above equations (1.1) and (1.2), *z* – Set of measurements given by the vector formulation,

$$z = \begin{bmatrix} z_1 \\ z_2 \\ . \\ . \\ . \\ z_3 \end{bmatrix} = \begin{bmatrix} h_1(x_{1,} x_2, ...x_n) \\ h_2(x_{1,} x_2, ...x_n) \\ . \\ . \\ . \\ h_m(x_{1,} x_2, ...x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ . \\ . \\ . \\ e_m \end{bmatrix} \tag{1.3}$$

$$z = h(x) + e \tag{1.4}$$

where,

h(x) is the nonlinear function relating measurements to the state vector

x is the state vector of the system

e is the vector formed by measurement errors

R is the covariance matrix in which the measurement errors are not correlated such that $E(e_i e_j] = 0$ and $E[e_i] = 0$ $(i = 1, 2....m)$.

The measurements are generally referred using variables *i* and *j*. In the equation (1.5), σ refers to measurement standard deviation.

$$R = \begin{bmatrix} \sigma_{11}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{22}^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sigma_{mm}^2 \end{bmatrix} \tag{1.5}$$

For the first order objective function *J(x)* to be at the minimum value the condition as given in (1.6) has to be satisfied. *H(x)* used in (1.6) can be elaborated as given in (1.8).

$$g(x) = \frac{\partial J}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \tag{1.6}$$

$$H(x) = \frac{\partial h}{\partial x} \tag{1.7}$$

$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \end{bmatrix} \tag{1.8}$$

Detailed expression of $H(x)$ is given in (1.8). A Taylor series expansion of the non-linear equation $g(x)$ around the operating point given by state vector x^k yields (1.9). On eliminating the higher order terms of the series from this equation, an iterative solution scheme is obtained and can be written as in (1.10),

$$g(x) = g(x^k) + G(x^k)(x - x^k) + \cdots = 0 \tag{1.9}$$

$$x^{k+1} = x^k - [G(x^k)]^{-1} g(x^k) \tag{1.10}$$

$$G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) R^{-1} H(x^k) \tag{1.11}$$

where k is the iteration index

x^k is the solution vector at iteration k,

$G(x^k)$ is gain matrix

The gain matrix $G(x^k)$ is sparse and positive definite for an observable system [12], [13]. The equation can be represented in terms of the change in state Δx,

$$\Delta x^{k+1} = [G(x^k)]^{-1} H^T(x^k) R^{-1} [z - h(x^k)] \quad (1.12)$$

Equation (1.12) is referred to as a normal equation. The initial state values will be mostly 1 per unit for voltage magnitude and 0 degrees for phase angle estimates which can be referred to as a flat voltage profile. With the initial assumptions for states of x, (1.12) is evaluated and the change in states Δx is obtained. The change value Δx gets added to the initial value of x followed by evaluation of (1.12) and the iterative process advances by finding a change in x. This is repeated iteratively until the largest state change given by infinite norm of Δx is less than the tolerance assumed or maximum iteration count is reached. After this, the final state estimate of x is obtained. Calculation of gain matrix $G(x)$, measurement jacobian $H(x)$ and measurement function $h(x)$ and solving (1.12) are explained in greater detail in [12].

In control theory, observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs [14]. The measure of observability of the SE of the system can be obtained by analyzing the H matrix which is expressed in (1.8). For the network model assumed in SE to be fully observable, there must be enough measurement to make a full rank H matrix. Also, this ensures a solution to (1.12) without problems due to numerical instability. The role of observability and efforts made to ensure observability for this project will be explained in Section 2.4.

Some of the research efforts in the area of state estimation are discussed here. A new type of SE based on a hierarchical structure is introduced in [15]. This paper emphasizes the use of distributed, parallel and integrated SE within global state estimation. In theory, three types of SE are applied separately and this paper claims that using a state estimation encompassing the three types of SE is better than applying each type separately.

The temporary failures and local visibility problems are overcome in this system wide state estimation. In [16], an attempt to embed the FACTS devices in to existing SE is done as the presence of FACTS devices has increased in recent times. Network observability and bad data analysis in such SE is also presented. Above all, attempts to bring in phasor measurements into state estimation has been an active research in recent times. More details on phasor measurements and the challenges in bringing these measurements into SE are discussed below.

1.4 Phasor measurement units

PMUs were developed in the 1980s and are now used for power system measurements in many parts of the world [17]. An important feature in PMU applications is the use of measurements with highly precise time stamps. This provides an approach to analyze and use the data at the same instant from geographically distant devices. This time stamping is provided by a signal from GPS.

The current and voltage inputs, which finally evolve as measurements, come from secondary windings of current and voltage transformers respectively. These inputs are fed as analog signals into anti-aliasing filters and the frequency response of these filters are dictated by the sampling rate chosen [18]. The outputs from these filters are converted to inputs of suitable range for analog-to-digital (A/D) converters. The digital output from A/D converters is passed onto a phasor microprocessor which estimates the positive sequence voltage and current signals. Along with voltage and current, frequency and rate of change of frequency signals are also provided from PMU.

The sampling clock is phase locked with a GPS clock pulse which is provided by GPS satellites. For PMUs, a clock pulse at intervals of every 1 second is important to synchronize the time sampling at different locations. PMUs are located at power system substations that are widely distributed. Data from several PMUs are received by phasor data concentrators (PDCs). PDCs have the facility to store the data in large amounts and then forward the data to control center [19]. In this entire process which involves extracting the signals from instrument transformers to receiving these signals at control center, there are numerous possibilities for errors to occur in these signals.

1.5 SE augmented with phasor measurements

There are many practical problems in using phasor measurements in state estimation. One of the most important problems is the presence of bad data among phasor measurements. Bad data can significantly alter the final state estimates resulting in poor performance of the SE and thus removal of bad data from PMUs is important. Effective bad data and topology error processing through optimal placement of PMUs is provided in [20], [21]. It is shown that the presence of bad data in critical measurements can be largely reduced by including few PMU units that are strategically placed.

A method to avoid the choice of the reference bus while using PMU measurements is presented in [22]. It is claimed that removing the reference bus helped to detect, identify and remove erroneous phasor measurement. In [23], an iterative algorithm to tune the PMU weights based on the state estimation results corresponding to the PMU is proposed. The beneficial impact on bad data detection due to the proposed tuning of PMU weights is presented in [24]. In the traditional state estimation, constraints on zero injection nodes and phasor measurements are added to improve its performance in [25]. The optimal estimation under the double constraints is done using the Lagrange multiplier method. In [26], introducing pseudo power flow measurements calculated using voltage and current phasor measurements instead of applying the latter directly is attempted. It is demonstrated that this technique facilitates smoother convergence while avoiding numerical instability problems as well as results in more precise state estimation. The number of analog channels and communication constraints varies with PMUs from different manufacturers. The algorithm described in [27] for optimal placement of PMUs includes the number of channels as a variable. It is demonstrated that strategic placement results in robust operation against loss of a single PMU. The various types of errors present in the phasor measurement signals along with methods to correct these errors are already analyzed to a greater extent in [28]. The intent is to remove these errors before using the data in state estimation.

Time skew occurs between measurements from different PMUs due to the presence of inaccurate time stamps. For example, if the reporting rate of a PMU is 30 measurements per second, the interval between every two measurements is 1/30 seconds. Due to the inaccuracy of the sampling clock if ε is the time skew component, the interval gets changed to $|(1/30) \pm \varepsilon|$ instead of 1/30 seconds. Due to this difference in interval, these measurements are not actually synchronized with the rest of PMUs. It was identified that the time skew error can be treated as a constant and a Kalman filter was proposed to correct the errors in the measurements resulting from time skew. In the present work, the method proposed in [28] to rectify time skew is adopted.

The number of measurements given as an output from any device can be termed as reporting rate. There is a remarkable difference between the reporting rates of PMUs and conventional SCADA devices. For a 60 Hz system, a PMU device reports 10 - 30 observations per second [29]. However, the conventional technology using SCADA delivers measurements for every 4 seconds [30]. The state estimator typically runs once in every 2 – 3 minutes and thus it is necessary to account for the difference in reporting rates between the two devices in order to utilize the data from both the devices [6]. A diagrammatic representation of SE as a process with the interval of 30 seconds is shown in Figure 1.1.

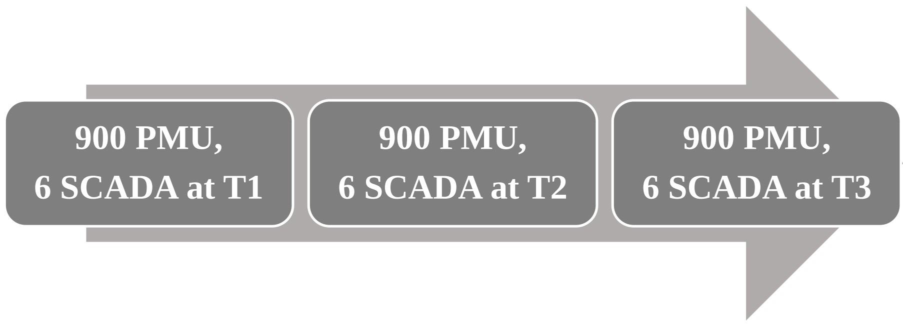

Figure 1.1 Diagram showing reporting rates of PMU and SCADA measurements

From Figure 1.1, it can be seen that there will be a large number of phasor measurements from the last instant of the state estimation to the present and hence choosing the best PMU measurement from the whole set is still a subject of study. Since the PMU data are not free of measurement error, it is difficult to choose a single measurement as the best possible measurement. The refresh rate of PMU and SCADA is taken as 30 frames per second and one observation per 5 seconds respectively.

The problem in choosing a single phasor measurement is due to the presence of inherent errors from various components within the PMU. To a certain extent, communication channels in instrument transformers contribute to these errors [31]. These errors contain systematic and biased parts as well as unbiased random noise. The biased errors can be rectified by methods proposed in [32], [33]. The unbiased errors due to random measurement noise are not straightforward to eliminate. A simple method to reduce the error in the data due to noise would be taking an average over a series of observations [34]. Here, the noise is assumed to be following an independent and identical distribution (*i.i.d.*). Lack of correlation between the observations defines the independent characteristic of the data. The characteristic for identical distribution is the whole series of observations is assumed to be following a single distribution, for example Gaussian [35]. This relation between noise reduction and averaging forms the basis for using a mean value of a buffer of phasor measurements rather than using a single measurement. The concept of using a buffer is shown in Figure 1.2.

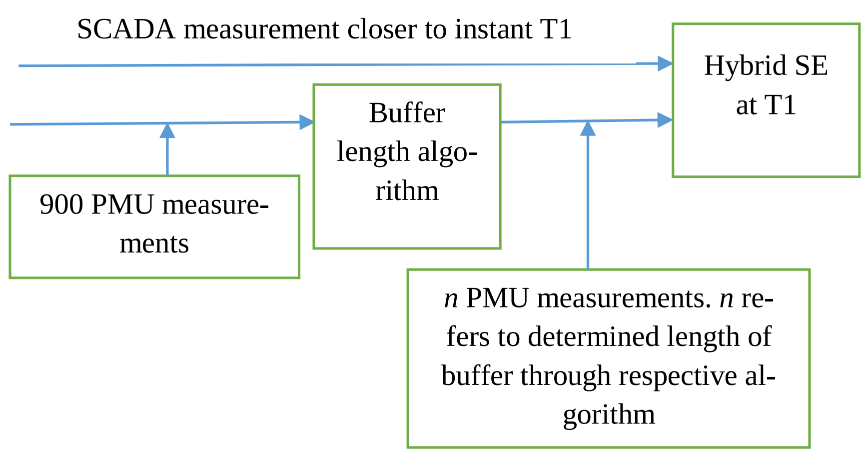

Figure 1.2 Conceptual diagram for phasor measurements buffer

The problem in designing a buffer for PMU measurements is the determination of the size of the buffer. A large number of phasor measurements in a buffer could bring a better noise reduction if the system is static. This could lead to an argument supporting the usage of all PMU observations from the last to the present instant of SE, say 900 phasor measurements as shown in Figure 1.1. In contrast, the power system is never truly stationary. A typical set of voltage magnitude observations from phasor measurements is shown in Figure 1.3 below.

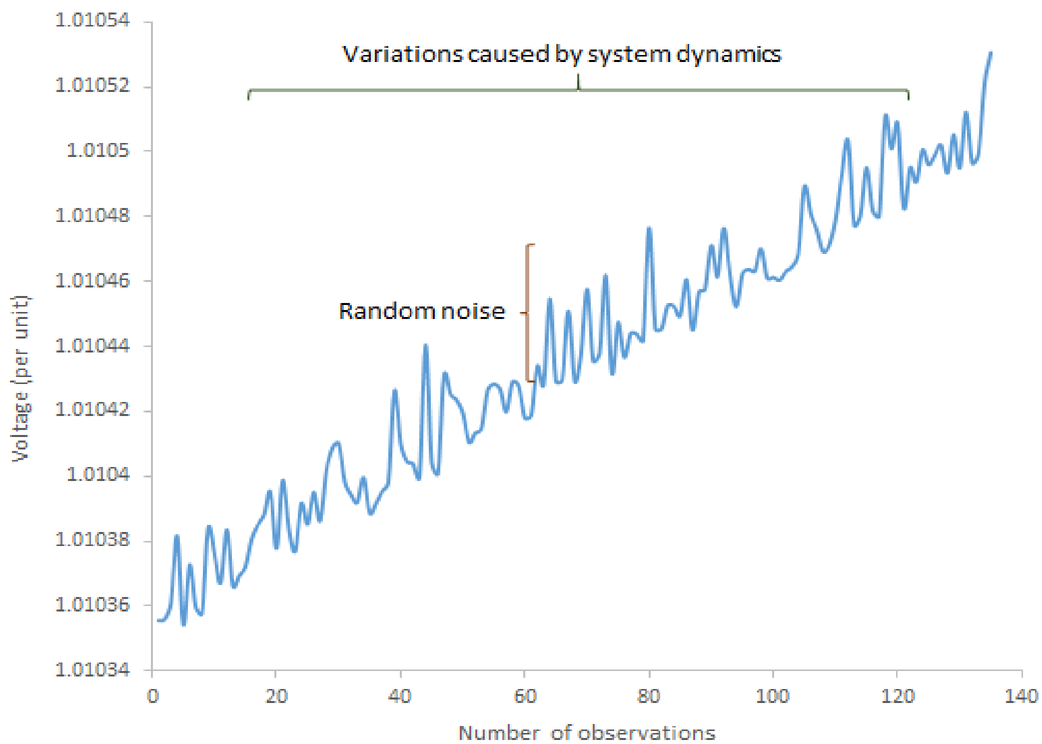

Figure 1.3 Sample observations from PMUs with noise and system dynamic

Here, the longer buffer length would include some system conditions from the past which could be significantly varied from the present. The inclusion of observations that are obsolete could distort the measurements at the present. These two contradicting aspects of PMU buffer pose a challenge in choosing the optimal length of the buffer. The first aspect is uncertainty due to noise and second aspect is variation of data due to system dynamics [36].

1.6 Research motivation and objectives

The conventional measurements obtained from SCADA that are primarily used in SE include branch active and reactive power flows, power injections (active and reactive) and bus voltage magnitudes. The contemporary trend is to augment these measurements with PMU measurements of *phasor* voltages and create a hybrid state estimator. The focus is to obtain the state estimation solution with and without phasor measurements, and to identify the combination that provides better state estimates. The key objective is to develop an algorithm to find the optimal size of the *buffered* phasor measurements. This algorithm would be used for every PMU and thus would result in *variable* buffer lengths of phasor measurements. Analyzing the benefits of using variable buffer length from the algorithm over using a fixed buffer length is needed.

There are limited studies on the impact of phasor measurements in state estimation using a practical system with real life data. This limitation forms the basis of this research project. The real time network information along with its data from a utility in southwest part of the USA is used. A set of indices are defined to evaluate the performance of SE after using phasor measurements. The same set of indices are used to show the improvement in using variable buffer lengths for PMUs. A buffer length algorithm was already developed and tested on a real life based test system in [36]. In order to support its testing, the same algorithm used in [36] will also be analyzed in the system considered in the present project and further investigation of the buffer length algorithm will be done. Attempts to develop new algorithms and using new statistical tools to enhance the results and analysis will be carried out in this research work.

1.7 Organization of the thesis

A brief review of state estimation, its components and importance of phasor measurement units in SE are given in the present chapter. Chapter 2 describes the real time data used in this project, customizing the data format for compatibility with the software used in this research, essential steps for state estimation without phasor measurements and results of SE.

In Chapter 3, incorporating phasor measurements into state estimation and various methods used for this are discussed. Chapter 4 states the techniques followed to gauge the improvement in results after using proposed methods and various figures demonstrating the benefits of hybrid SE and variable buffer lengths.

Chapter 5 concludes the thesis and enumerates the contributions of this research. Finally, the possible future work in this field is also discussed.

Appendix A contains a sample of the data and measurement format used in the project. A portion of MATLAB codes developed in this research are provided in Appendix B. Detailed comparison tables for some of the results shown in Chapter 5 are given in Appendix C.

Chapter 2. State estimation without phasor measurements

2.1 State estimation using software - MATPOWER

MATPOWER is a software package based on MATLAB and it is used for solving power flow, optimal power flow and state estimation algorithms [37]. Throughout this project, MATPOWER is used for running state estimation for various cases. Measurements needed for state estimation are made available in Microsoft Excel and retrieved into MATPOWER as input. This is preferred for flexibility in dealing with large volumes of data. The system information containing buses, branches and generator is provided in a MATLAB (*.m) file. The basic version of the program is readily available and it can be modified by a user as and when required.

There are few modifications needed to the initial code. The changes are required to:

- accommodate phasor measurement units
- include various buffer length algorithms as well as manually provided buffer length values
- assign different standard deviation for every measurement
- handle observability requirement
- enable usage of injection measurements
- calculate residuals.

2.2 Data received from the utility

Work represented in this document used actual functioning large scale power systems as test beds. These test beds were part of the Western Electricity Coordinating Council system. Actual measurements were used wherever possible. The input measurements and output solutions of SE data along with their corresponding network information are received from the utility. These are real time data from the actual utility environment. A total of five instances of these sets of data were received. Each set represents a different loading condition during the daily operating horizon. This enables the examination of the performance of the proposed approach at different load levels. The idea is to use SE data from the utility and to arrive at state estimates reasonably close to the SE solution from the utility. This is needed to support the claim that the improvement in state estimates through the methods proposed in this research can be realized by the actual state estimator used in the utility. Figure 2.1 shows load condition for some of the instances of data.

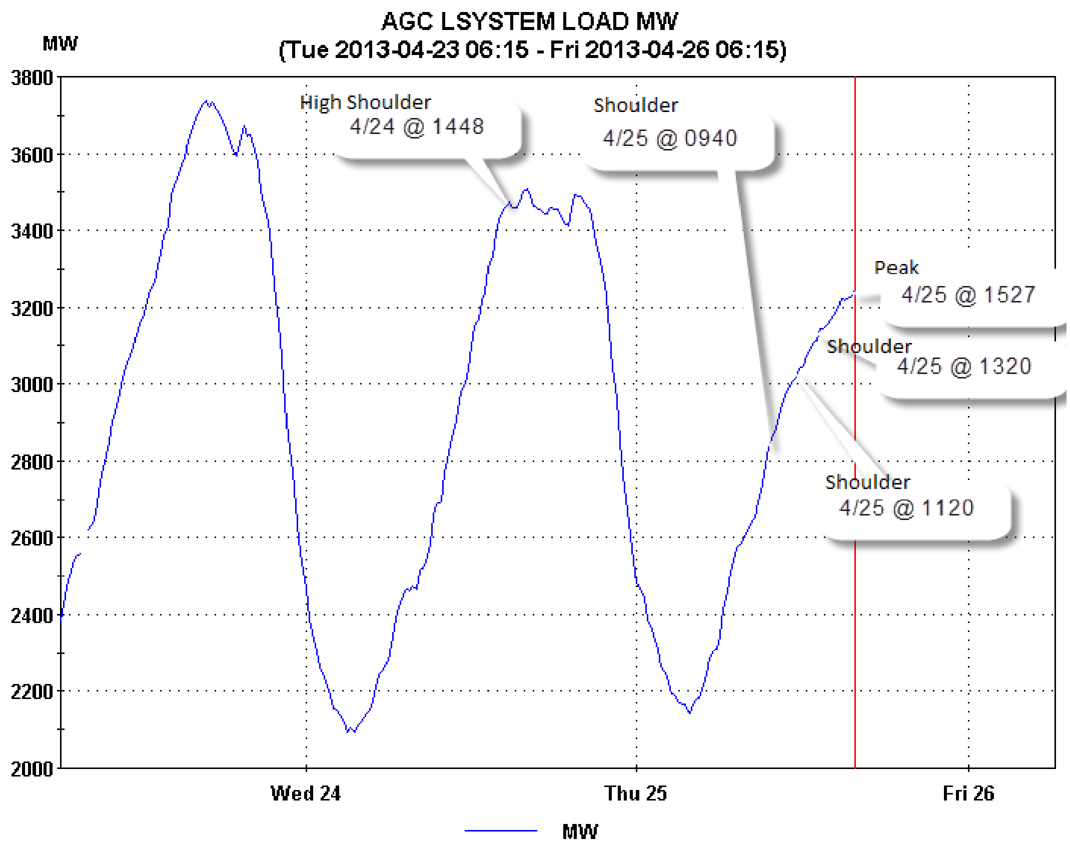

Figure 2.1 Different load conditions for the data sets

All the 5 datasets contain bus, branch and generator information needed to represent the whole connected network, the input measurements and state estimation solution. In addition, 1 hour of phasor measurements going back in time from the present instant of each SE data are provided. There are two data sets at the 'shoulder' load level and one data set each at the rest of the three load levels. Reference to the respective load levels using the set numbers mentioned in Table 2.1 is followed throughout the document. The details of load conditions and its corresponding set number can be seen in Table 2.1. All the datasets in Table 2.1 can be seen in Figure 2.1 except the set 5 corresponding to 'low shoulder' level.

Table 2.1 Set number and their respective load levels

Set	Load condition	Time stamp
1	High shoulder	04/24 @1448
2	Shoulder	04/25 @0940
3	Peak	04/25 @1527
4	Shoulder	04/25 @1120
5	Low shoulder	04/26 @0800

2.3 Customizing the data format and the program

The data is received in the format followed by the utility. It is mandatory to make changes to the data format before passing the data as input into MATPOWER. All the system data hold external numbers for bus, branch and generator. Conversion from external to internal numbering is automatically carried out by MATPOWER.

Branch

The connected system is represented mainly through branches. Topology status, referred as ‘tp_status’ in the data, is numbered from 0 to 3 where each number refers to a particular branch connection as explained in Table 2.2.

Table 2.2 Topology status – type and its numbering

Numbering	Connection type
0	Outage
1	ON
2	Open ended
3	SE – closed

All the branches with topology status 0 and 2 are removed from the input. The intent is to remove all kinds of unconnected network information from the input system data. There are few branches with invalid bus numbers and with same from and to bus numbers. These invalid branches are removed to obtain the final set of input branch data.

The key fields in the branch data that are required as input are

- From bus
- To bus
- Resistance (r_{br})
- Reactance (X_{line})
- Fixed shunt (S)
- Tap ratio of transformer
- Phase angle shift from transformer

- Status.

Bus

The bus information should follow the branch connection. This means only the buses present in the filtered branch data are considered for the bus input. A program written for this purpose is given in Appendix B.1. The key fields in the bus data that are needed for the input are

- Bus number
- Bus type
- Load active power (P_L)
- Load reactive power (Q_L)
- Shunt active power (G_s)
- Shunt reactive power (B_s)
- Phasor voltage magnitude (V_m)
- Phasor voltage angle (V_a)
- Area and zone number

Generator

The topology status field in the generator data takes the value either 0 or 1. The value 1 represents generators in operation and thus only those generators are considered for input. The data for some of these generators are represented in multiple entries containing only a fraction of their capacities. This means the multiple entries contain the same information except that only a part of active and reactive power capacity is reflected. This would give an unclear picture to the software and thus all such entries for the same generator are put together into one entry with whole capacity. Generator information present in the input are

- Bus number of the generator
- Generator active power (P_G)
- Generator reactive power (Q_G)
- Maximum reactive power limit (Q_{max})
- Minimum reactive power limit (Q_{min})
- Maximum active power limit (P_{max})
- Minimum active power limit (P_{min})
- Scheduled voltage ($V_{scheduled}$).

Measurements

As discussed in Section 1.2, bad data detection and removal is important to ensure better state estimates. In this work, offline study of SE is currently done using the data from past and hence both the estimates and measurements are available. The SE is done using commercial software in the utility environment and it is assumed to have provided reliable estimates. Therefore, normalized residuals can be used for identifying bad data. These normalized residuals are the difference between measurements and estimates divided by their respective standard deviation of the measurements [38]. The larger normalized residuals reflect that the corresponding measurement is not accurate and hence the measurement is removed from the input. In this research, the bad data removal is employed by a heuristic method. After many attempts of SE, it is observed that the normalized residual values between - 4 and 4 reflects corresponding measurements to be more accurate. The measurements with residuals outside this range represents represent outliers that should be discarded. Rejecting these outliers filters reliable measurements for both active and reactive power flow. For active and reactive power injection measurements, only those measurements with a normalized residual in either a range of - 4.5 to 4.5 or - 4 to 4 are considered for input. These rejection procedures are justified because the percentage of normalized residuals in absolute value beyond 4 in a Gaussian set with unit variance is very small or negligible. This means that the detected residuals are outliers and do not follow the assumed Gaussian distribution. At this point, the set of input measurements and appropriate network model are available to execute SE. Furthermore, there are few more essential steps to be followed to obtain SE solution closer to the one from the utility. A sample of the input system data and measurements data format can be seen in Appendix A.1 and A.2 respectively.

2.4 Essential steps for SE

The measurements that are appropriate for SE can be identified by their respective measurement standard deviation (MSD). Only the measurements that have the corresponding non-zero MSD value are considered for input measurement set. This applies to both power flow and injection measurements. This is because the standard deviations are used to create the covariance matrix as given in (1.5) and numerical instability will occur in the presence of zero standard deviation value when the R is inverted as per (1.12). More specifically, the injection measurements with standard deviations as zero shows that the corresponding buses are unobservable.

Observability is very important to obtain a unique estimate of all the states [39]. Thus, the states corresponding to those unobservable buses are removed from the estimation process in order to obtain unique state estimation. This helps to ensure full observability for the states of interest and facilitates a smooth solution of (1.12). However, these buses are present in the system data to ensure the integrity of the network information.

The initial value of the states is one of the deciding factor for the speed of convergence towards the final solution and there are two ways of defining the values of the initial states. The first approach is to provide state estimated values of the previous SE instant as the starting value of states. The other approach is following a flat voltage profile. For all sets of data, the solution received from the utility is provided as the initial value of the states. This is consistently followed in both conventional and hybrid SE. Even the voltage magnitude states corresponding to generator buses are provided with the estimates from the utility. It is learned that the general trend in the utilities is to follow this approach rather than the flat voltage profile.

2.5 Comparison of SE results

The aforementioned steps are essential to ensure that the SE results from MATPOWER are reasonably close to the SE solution from the utility. It is ensured that around 99 percent of the voltage magnitude estimates from MATPOWER SE are within ± 0.05 per unit from that of estimates from utility for all the sets. Relative phase angle estimates are also compared in a similar fashion as voltage magnitudes. It is seen that around 99 percent of the phase angle states from MATPOWER SE are within ± 2 degrees from that of angle estimates from utility. This can be observed for all the sets. Scatter plot for voltage magnitudes and relative phase angles comparison are given in Figures 2.2 to 2.11. The number of voltage magnitudes and phase angle estimates falling within the specified range varies for every set of SE data. The total buses whose states meet this condition can be seen from Table 2.3 and Table 2.4.

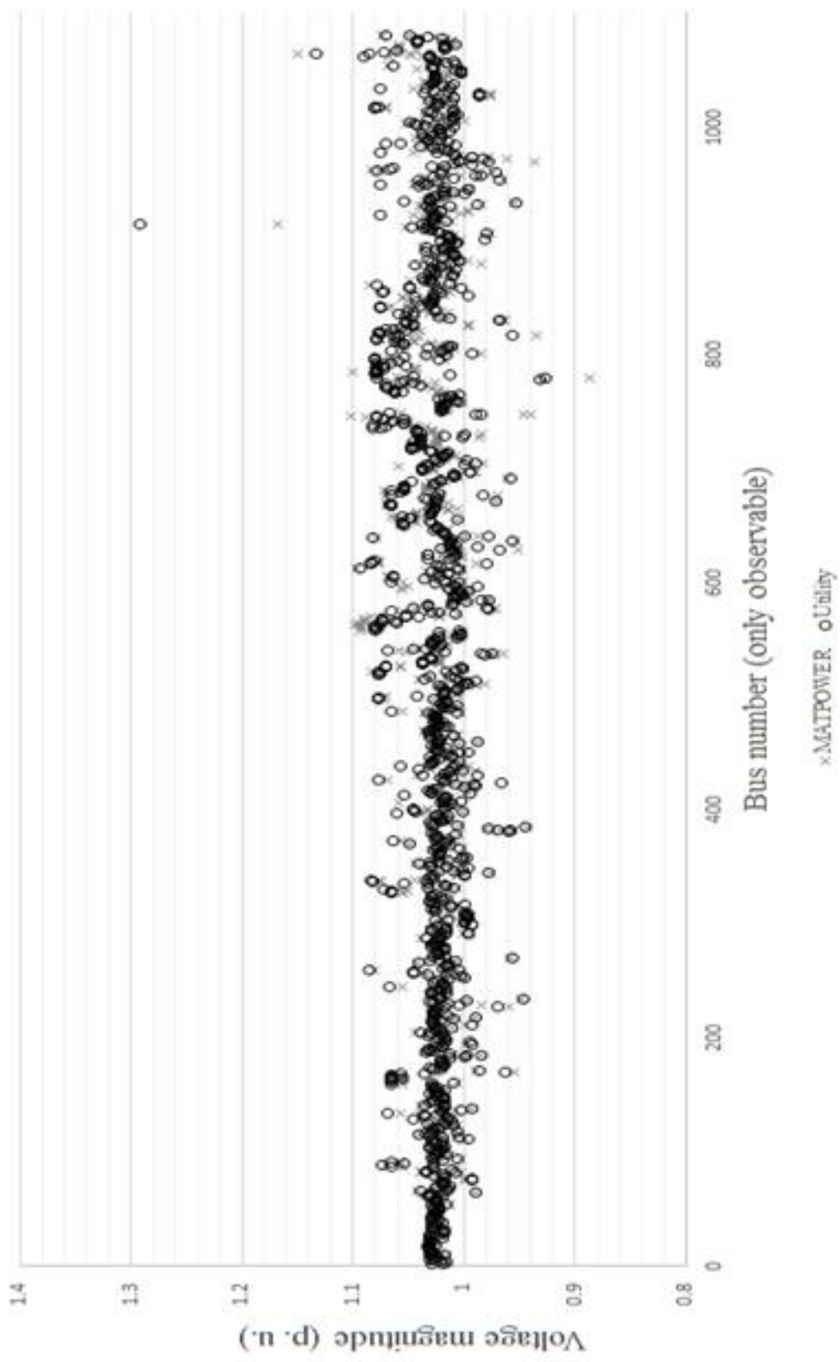

Figure 2.2 Voltage magnitude estimates from MATPOWER and utility - set 1

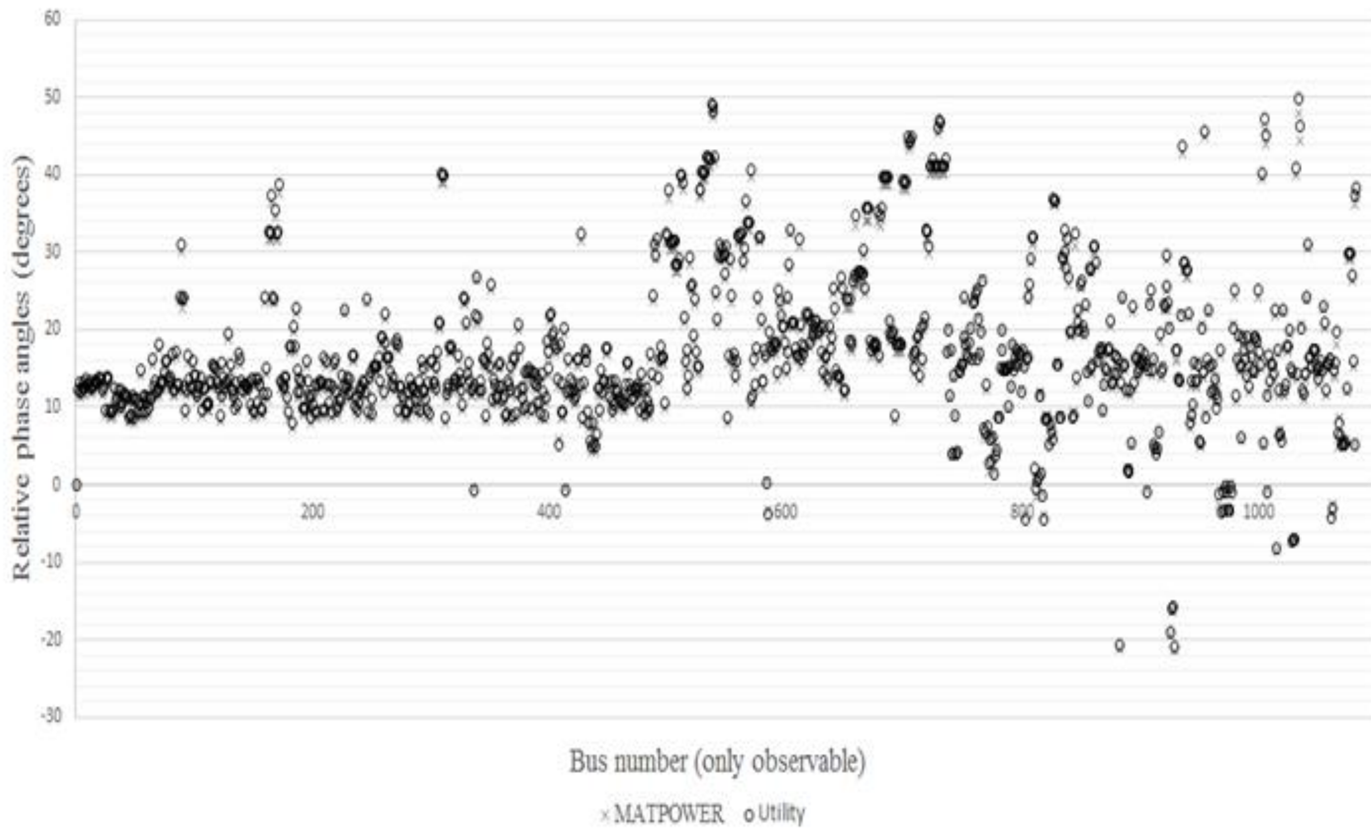

Figure 2.3 Relative phase angle estimates from MATPOWER and utility - set 1

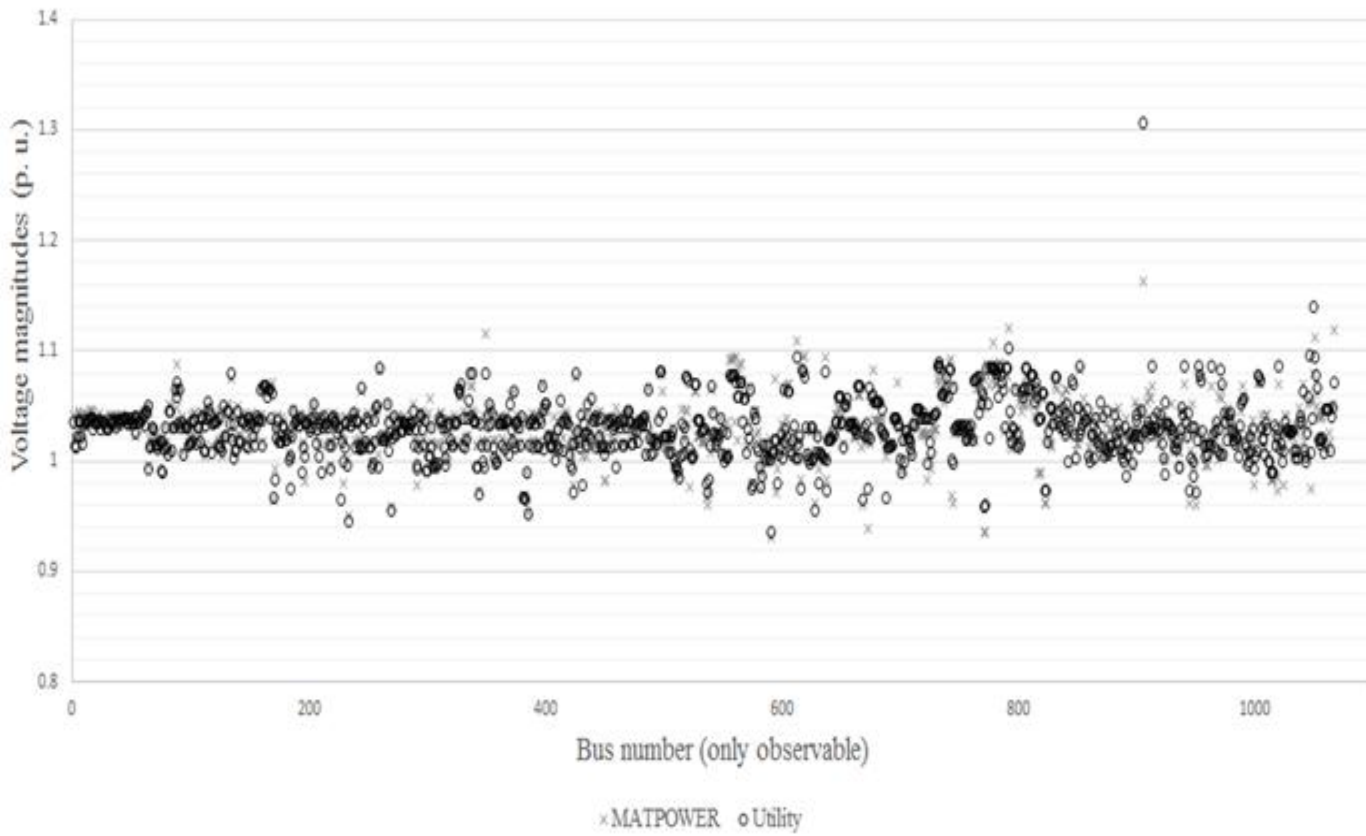

Figure 2.4 Voltage magnitude estimates from MATPOWER and utility - set 2

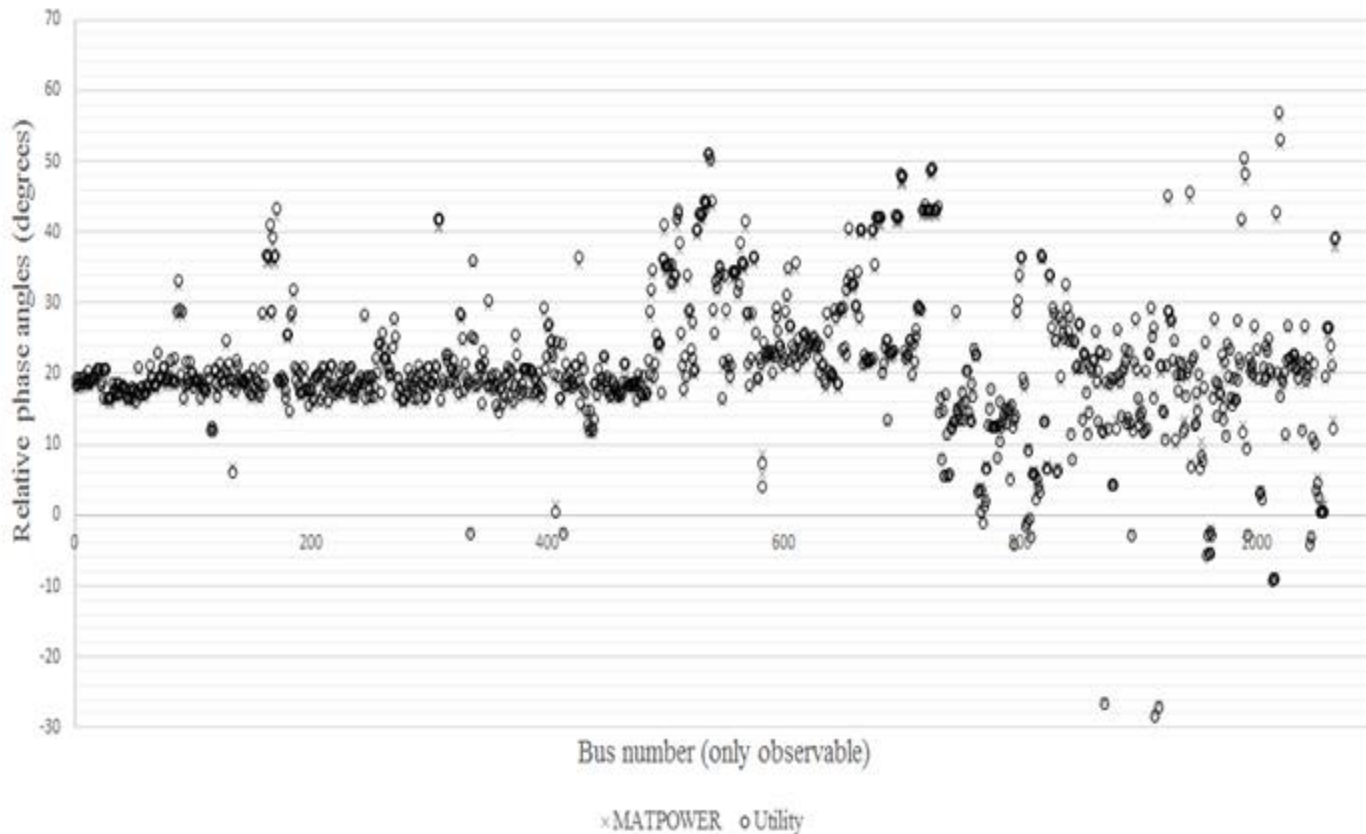

Figure 2.5 Relative phase angle estimates from MATPOWER and utility - set 2

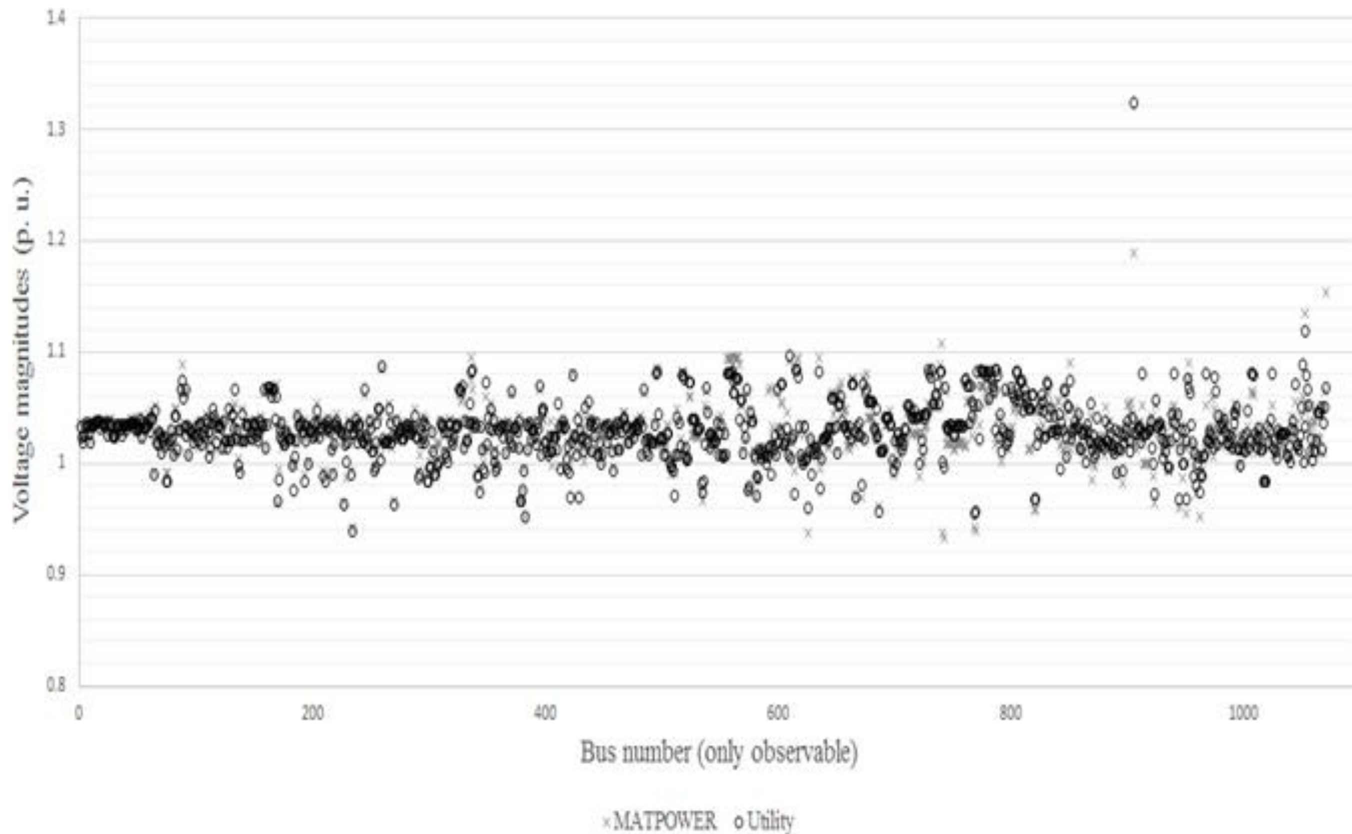

Figure 2.6 Voltage magnitude estimates from MATPOWER and utility - set 3

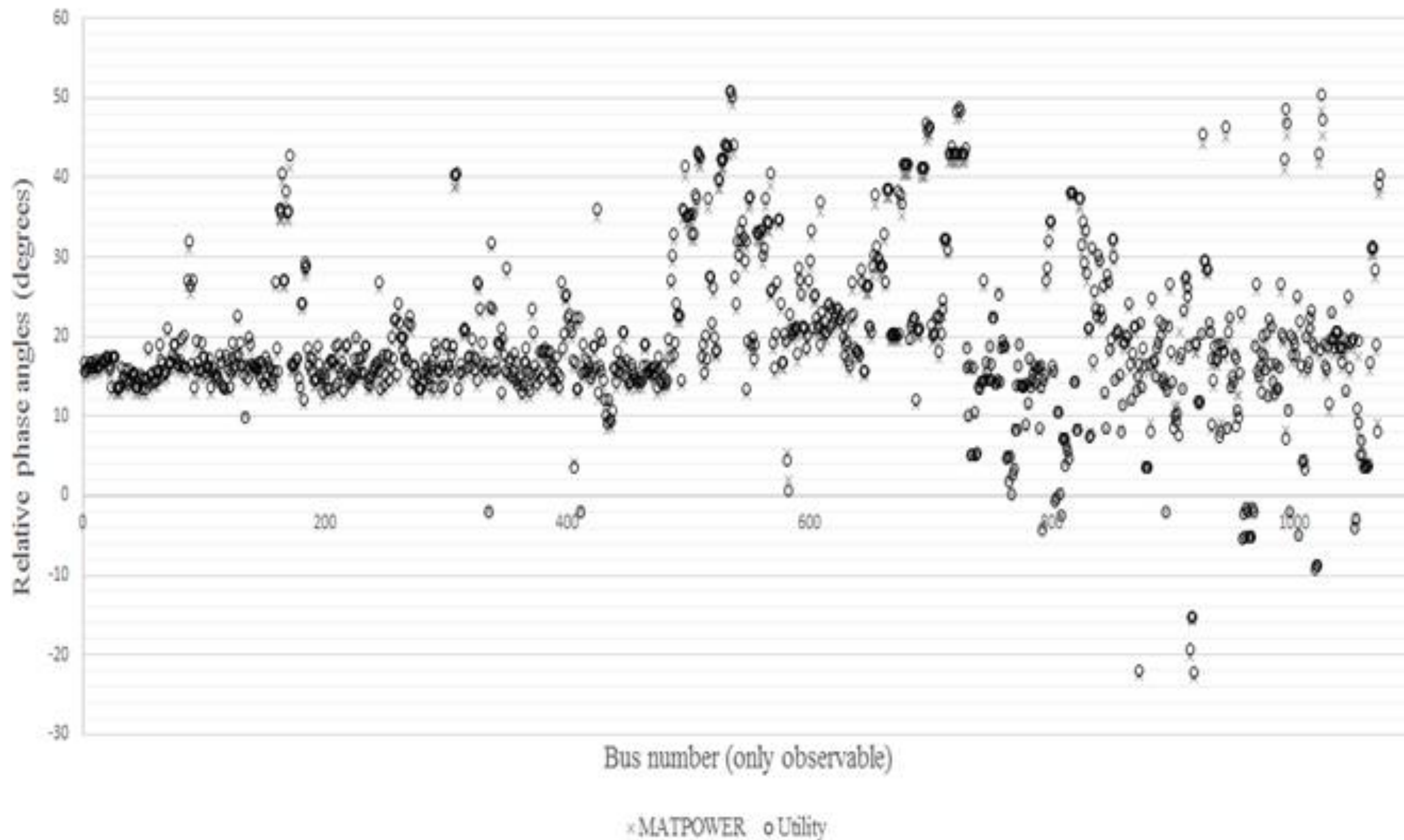

Figure 2.7 Relative phase angle estimates from MATPOWER and utility - set 3

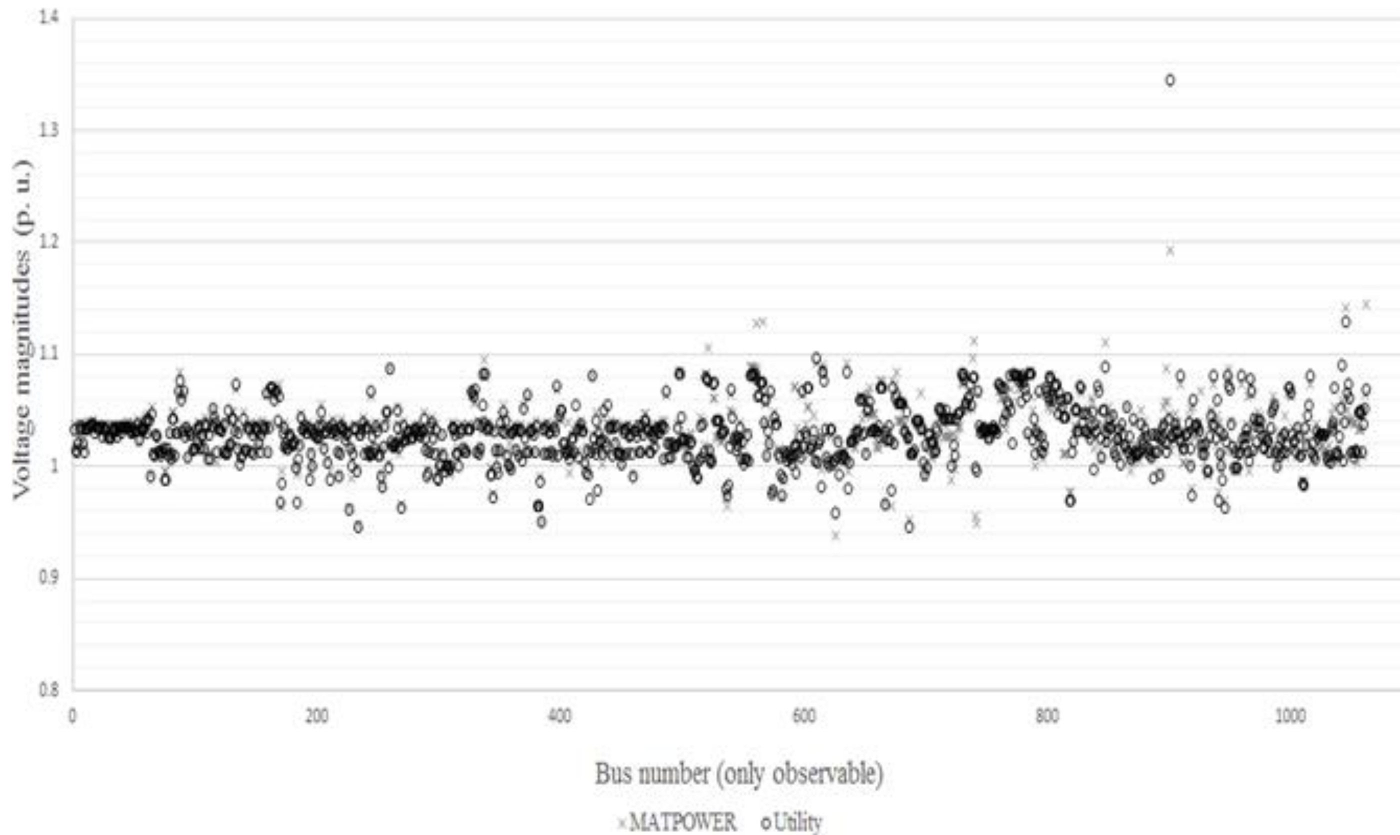

Figure 2.8 Voltage magnitude estimates from MATPOWER and utility - set 4

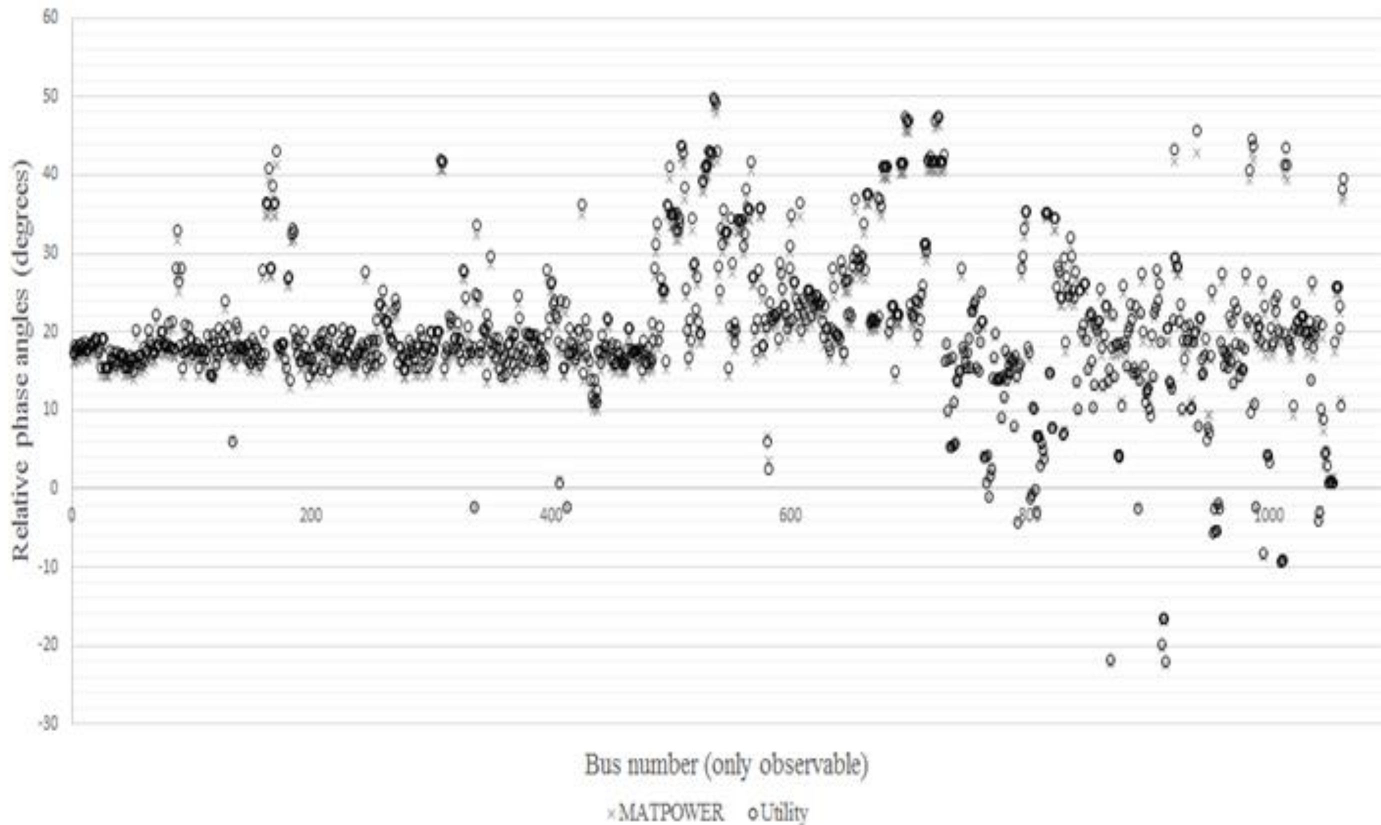

Figure 2.9 Relative phase angle estimates from MATPOWER and utility - set 4

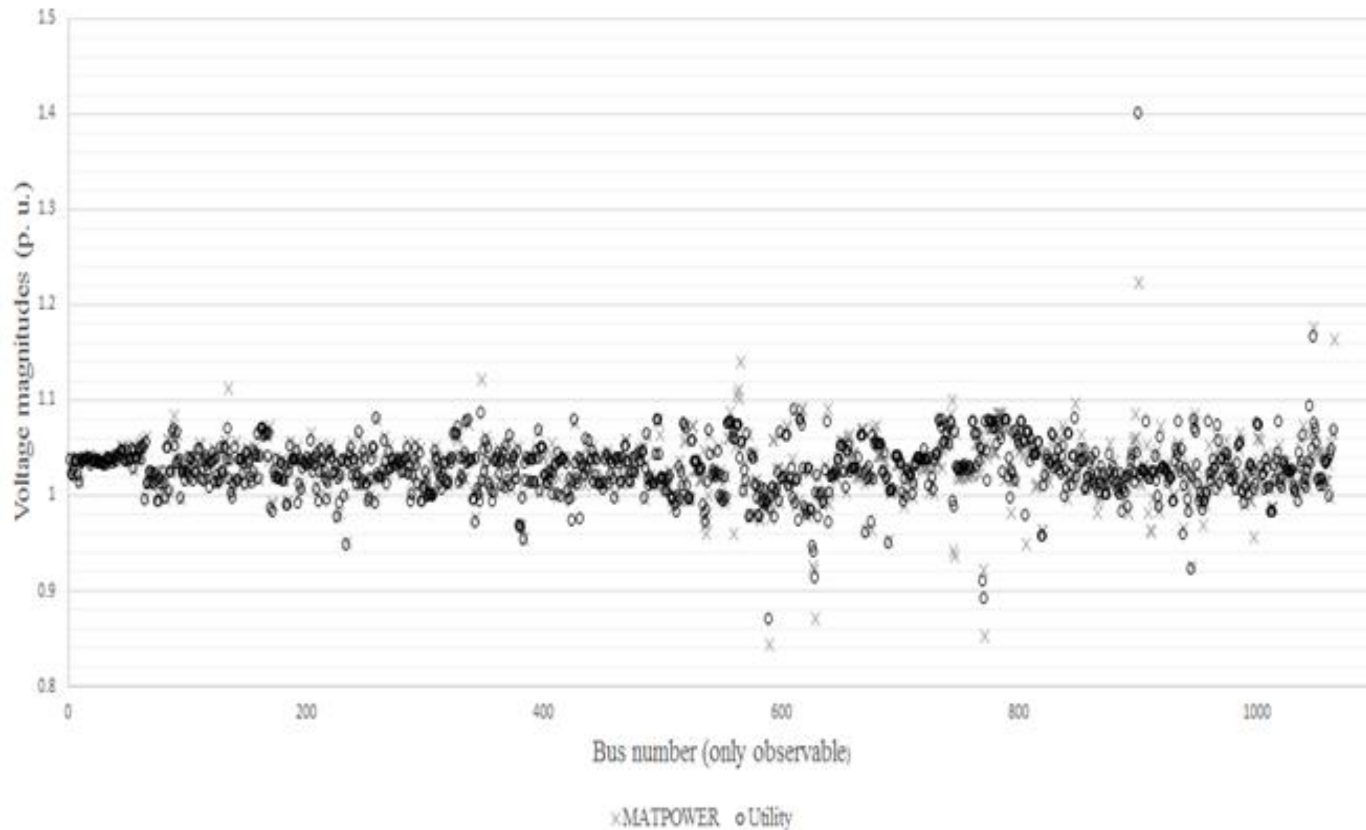

Figure 2.10 Voltage magnitude estimates from MATPOWER and utility - set 5

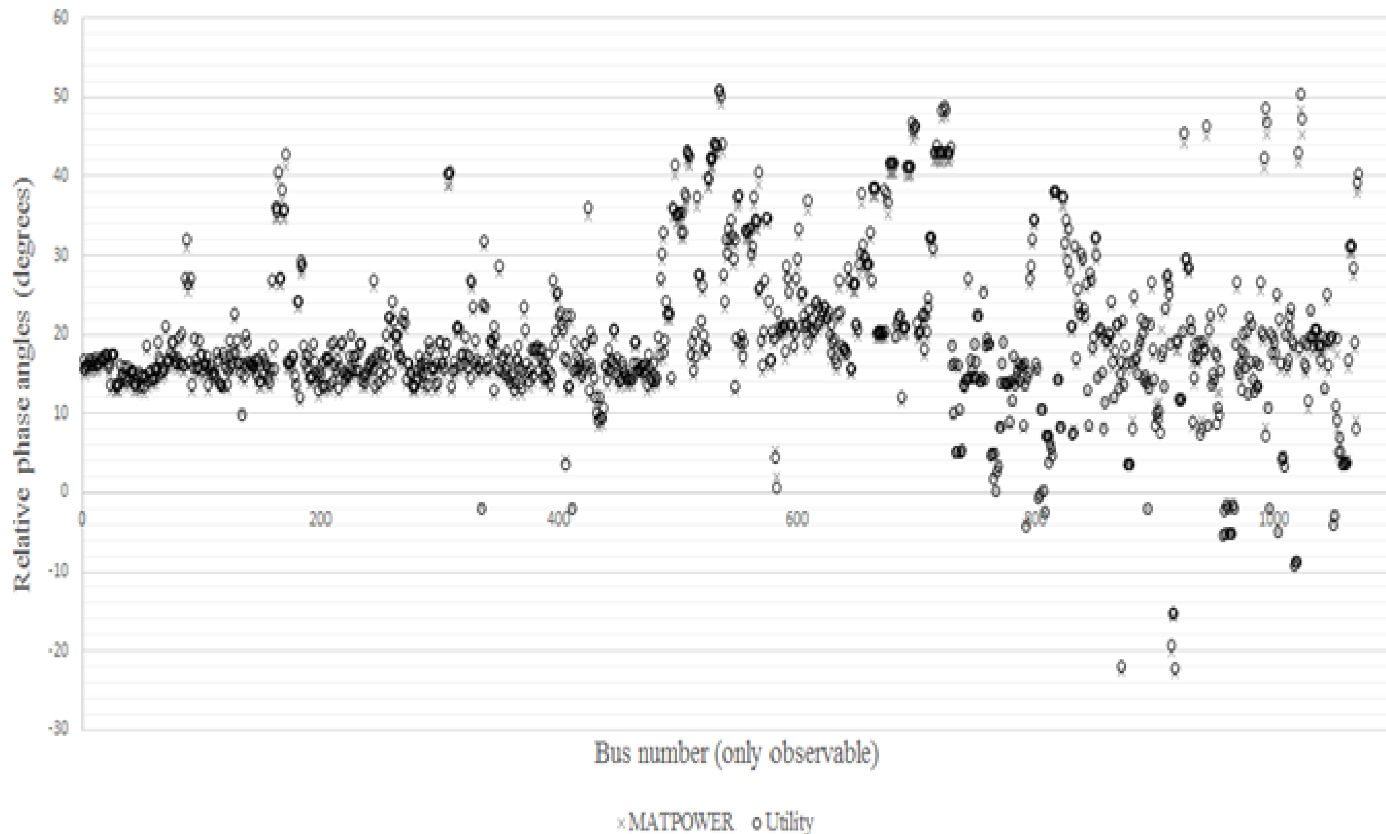

Figure 2.11 Relative phase angle estimates from MATPOWER and utility - set 5

Table 2.3 Voltage magnitude estimates within the range ± 0.05 p. u.

Da-taset	Total number of buses observable	Number of buses within the range	Percentage of buses within the range
1	1081	1080	99.91
2	1066	1057	99.16
3	1070	1060	99.07
4	1062	1055	99.34
5	1064	1050	98.68

Table 2.4 Relative phase angle estimates within the range ± 2 degrees

Da-taset	Total number of buses observable	Number of buses within the range	Percentage of buses with the range
1	1081	1080	99.91
2	1066	1057	99.16
3	1070	1060	99.07
4	1062	1055	99.34
5	1064	1050	98.68

2.6 Brief summary of the SE results

The comparison of the SE results indicate that a reasonable percentage of the buses have their state estimates within the specified range from the utility SE solution. Although not all the buses are within this range of the utility solution, this result is justified because of four reasons. They are

1. The state estimator in the utility is run as a two-step process. The first step keeps the voltage magnitude and power injections constant at observable buses and runs the power flow (PF) for the entire system using pseudo measurements for the remaining unobservable buses. The branch power flows calculated between the unobservable buses from the PF solution are used as pseudo branch power flows. In the second step, actual state estimation is run using these pseudo power flows and measurements at observable buses. This is different from the SE process done in this study.
2. Utility SE uses state estimates from the previous instant as the initial estimates of states whereas MATPOWER SE uses the present instant of estimates from the utility solution as the initial value of states. This is likely to create difference in the final estimates between two solutions.
3. The techniques followed for bad data detection and removal in utility SE is assumed to be more robust.
4. Utility SE uses commercial state estimators available in the market. The backend implementation of SE and the assumptions adopted by this commercial SE is not known. Therefore, it is possible that the exact results as the utility SE is not likely to be achieved.

Although the estimates from Matpower SE are not as exactly the same as utility SE, these estimates are well within the desired range from the utility SE. Also, demonstrating improvement in state estimates using the hybrid SE is the main objective of this research and the SE results discussed above is good enough to proceed towards the objective. Details about this hybrid SE and its impact will be discussed in the next chapter.

Chapter 3. State estimation with phasor measurements

3.1 Phasor measurements into SE

Performing SE without phasor measurements has been accomplished at this point. The next step is to include phasor measurements for every set and evaluate if there is improvement in state estimates. In order to include phasor measurements in SE, the voltage magnitudes and relative phasor angles are added as input measurements at the appropriate buses. The data files for 31 PMUs are available with each file containing 1 hour of phasor measurements going back in time to every instant of SE data. However, only those measurements related to the SE solution in the data are required for analysis. It is understood that SE is performed every 30 seconds and phasor measurements are obtained as 30 frames per second. Therefore, the total number of phasor measurements relevant to every SE solution is 900 measurements accumulated prior to the instant of SE. The same number of measurements are fetched from the data files. The MATLAB script developed for this purpose is given in Appendix B.2. Throughout this discussion, the measurements are numbered in the order in which they are encountered while going back in time from the present SE instant. Consequently, the first measurement refers to the 900th measurement in the phasor measurements set relevant to the present SE.

Every PMU has its phasor measurements stored in individual files. The names of these files are stored separately and they are important in retrieving the measurements from each file. All the files have time stamps along with the measurements. Before fetching the measurements, the time stamp of the first data to be retrieved is verified with the time stamp of the SE. If the time stamps match, then the required number of measurements including the first measurement used for crosschecking are retrieved. This set of phasor measurements is treated as the buffer and processed through the buffer length algorithm. The output of this algorithm gives the number of measurements to be considered. The mean value of this specific number of measurements is taken as the final measurement from the PMU. The standard deviation of the same set of measurements gives the corresponding MSD. In case the standard deviation value is calculated as zero for a buffer, the average standard deviation from the rest of PMU buses is used. The above defined procedures are same for both voltage magnitude and relative phase angle measurements and the same steps are repeated for every PMU bus.

3.2 Methods for determining optimal buffer length

Three different algorithms to figure out optimal buffer length are used separately to evaluate the algorithm that gives more robust improvement with PMU. This is repeated for every set of SE data. The first algorithm, henceforth referred as *Method - I*, was originally developed in [36] and tested with a ‘real life’ based SE. With slight enhancement, this method is used in this work to test its performance in real time SE with actual measurements. The second algorithm, henceforth referred as *Method - II*, is developed and tested in the present work. The third algorithm, will be referred as *Method - III*, is utilizes a commonly used statistical tool known as R to arrive at optimal buffer length values. *Method – III* is closer to a process than an actual algorithm. All these methods work on the basis of finding mean and variance shift. The system dynamics would cause the mean value to change and thus mean shift detection is mandatory to extract the elements that does not reflect the system changes. Removing the errors due to noise is dealt through variance shift. The point at which the errors due to noise or other reasons is more than the specified limit is detected through variance shift evaluation. While processing every measurement or set of measurements, if either the mean or variance shift is found to be more than their respective thresholds, the processing stops and the buffer length value is obtained. The requirement to use both the mean and variance shift detection can be well supported from Figure 1.2. These three methods differ widely in the technique followed for variance shift detection as shown in Figure 3.1.

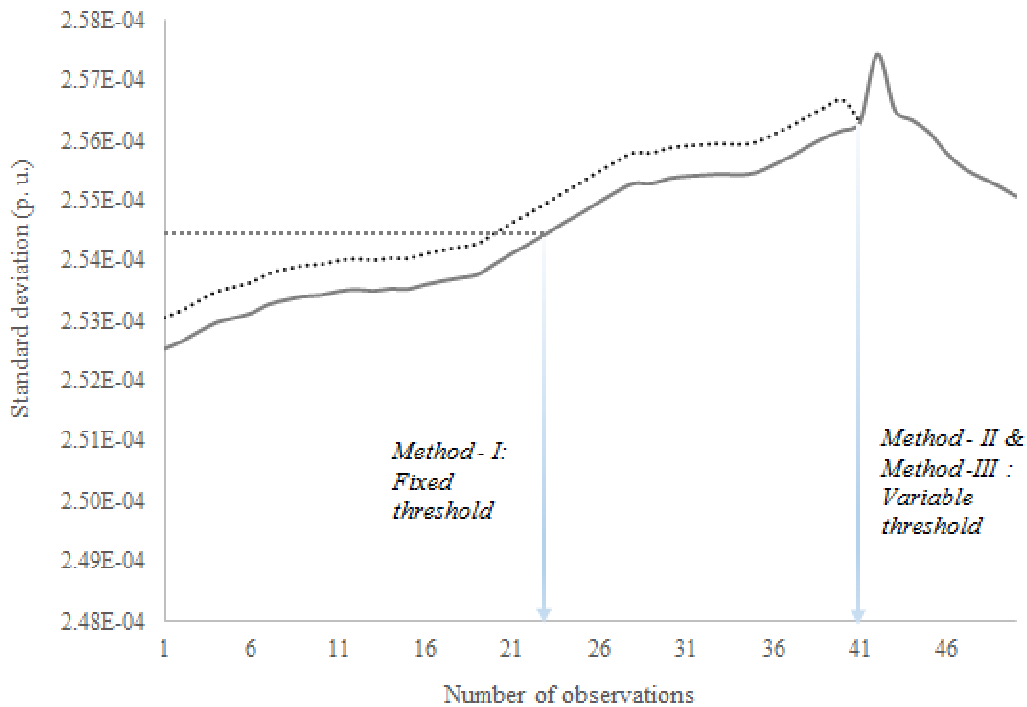

Figure 3.1 Diagram showing variance shift detection in three methods

Method – I

This is the simplest method of all the three proposed methods. A threshold standard deviation for identifying variance shift and hypothesis testing for mean shift are used for finding the buffer length. The threshold standard deviation values for voltage magnitude and relative phase angle measurements are set as 1e-4 per unit and 1e-3 degrees respectively. These threshold values are obtained heuristically by analyzing the standard deviation of a number PMU measurements. In this algorithm, the whole set of measurements from a PMU is divided into major subsets of size 30 each. These subsets are further divided into minor sets of size 5 and their standard deviation is compared with this threshold. This represents the variance test. In addition, the subsets are compared with each other for mean shift through hypothesis testing [40]. The standard ‘ttest2’ method from MATLAB is used for hypothesis testing. The arguments for ‘ttest2’ method are two sets of data, significance level, type of test (two tailed or one tailed or both), and variance type (equal or unequal) [41]. The two data arguments for ‘ttest2’ is provided from the minor sets of size 5 and default parameters are used for the rest of the arguments. This is sequentially repeated until all the subsets of 5 within the major subset is over. The same procedure is repeated for all the major subsets. A simple demonstration of this algorithm with the actual values is presented in the Figure 3.2. In the Figure 3.1, the letters A to E represent minor sets each of size 5 and h_1, h_2, h_3 and h_4 represent a series of hypothesis tests. The above algorithm is enhanced in this present work and can be seen below in the Figure 3.3.

A	**B**	**C**	**D**	**E**
25.0159	25.0162	25.0158	25.0157	25.0166
25.0160	25.0162	25.0156	25.0154	25.0161
25.0161	25.0161	25.0166	25.0153	25.0157
25.0162	25.0160	25.0163	25.0171	25.0153
25.0162	25.0159	25.0160	25.0169	25.0147

h_1 (A, B) h_2 (B, C) h_3 (C, D) h_4 (D, E)

Figure 3.2 Original algorithm for *Method - I*

A		B		C		D		E
25.0159		25.0162		25.0158		25.0157		25.0166
25.0160		25.0162		25.0156		25.0154		25.0161
25.0161		25.0161		25.0166		25.0153		25.0157
25.0162		25.0160		25.0163		25.0171		25.0153
25.0162		25.0159		25.0160		25.0169		25.0147

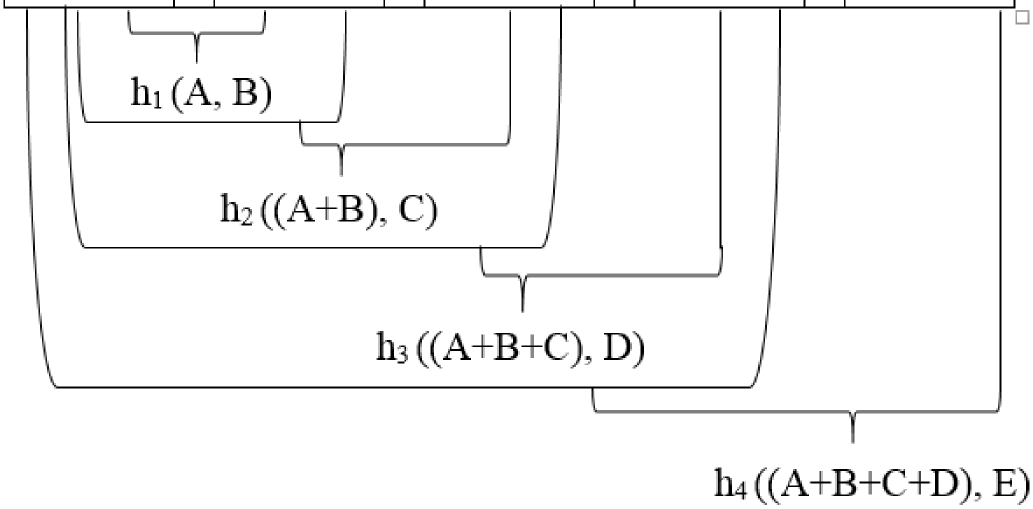

Figure 3.3 Enhanced algorithm for *Method - I*

With the new approach, the first hypothesis test h_1 stays same, however, the second hypothesis test combines the two minor sets (A and B) and uses as a single minor set for testing mean shift with respect to third subset (C). After this step, the first three minor sets (A, B and C) are grouped together and compared for mean shift against fourth minor set (D). The same technique is repeated for all the minor steps till the end of minor sets is reached.

Method – II

The formulation of a second method as well as implementation is done in this research. This algorithm is similar to *Method – I* in terms of using the mean and variance shift to detect any non-stationarity and find the desirable buffer length. However, this algorithm uses a series of threshold values for every arriving measurement in order to detect the mean or variance shift as opposed to *Method - I* with fixed threshold value. A statistical formula is used to update the threshold value for every element in the buffer beginning from the second to the last. The first measurement is always included in the buffer. Notice that the standard deviation cannot be estimated for the first measurement (only one measurement available) and thus a certain standard deviation or weight should be assumed for SE in cases of buffer length equal to 1. The statistical formulas and shift detection principle used are explained below. Consider *n* as the size of the elements of interest. The sample standard deviation (*sigma*) of the *n* elements calculated by (3.1) follows a chi-square distribution with *n -1* degrees of freedom. In (3.1), *X* is a random sample with mean $\overline{X}$. The variance of the sample standard deviation is given by (3.2) [42],

$$sigma = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 \tag{3.1}$$

$$V_n = 2(\frac{n-1}{2} - \frac{\Gamma^2(n/2)}{\Gamma^2((n-1)/2)}) \tag{3.2}$$

where Γ represents the inline gamma function in MATLAB,

$$std\_thr = sigma\sqrt{\frac{V_n}{n-1}} \tag{3.3}$$

In (3.3), the standard deviation variable s*igma* takes the value of *init_std* for the first calculation of *std_thr.* The *init_std* is the initial standard deviation while processing all measurements for a PMU. Initially, the standard deviation of first two measurements under process would be used to determine the initial standard deviation and if this value is zero, measurements are added incrementally till the standard deviation is non-zero. On encountering anon-zero initial standard deviation, it is assigned to the variable *init_std*. For all other value of *n*, *sigma* will be assigned the value of standard deviation obtained from *n - 1* measurements. Irrespective of the value of sigma, (3.3) is used to find *std_thr*. Equation (3.4) uses both *sigma* and *std_thr* to evaluate the threshold for detecting variance shift. Equations (3.1) to (3.4) is repeated for every size of the buffer and the upper threshold values calculated for every *n* is stored in *limit_array*. This processing is only needed to create threshold limits of standard deviation. The actual processing of buffer starts only after this step is completed.

$$upperthreshold = sigma + 3 * std\_thr \tag{3.4}$$

It is to be noted that theoretically the 3-sigma rejection rule, where the *upperthreshold* calculated in (3.4), is more accurate in a large set (*n* large) since the chi-square tends to the Gaussian distribution. For small values of *n*, a table derived from the chi-square distribution quantile can be used in (3.4). Only the upper threshold was used since the major concern here is to reject data which has jumps or shifted variance.

After the creation of *limit_array,* actual processing of the buffer starts. Now, the mean value of the present buffer (M_{buff}) is used to be compared against the mean shift threshold created by mean value of the buffer from previous step (M_{prev}). The H_{thr} and L_{thr} are upper and lower thresholds for detecting mean shift. Calculating these limits are shown in (3.5) and (3.6). In these equations, the three times the standard deviation away from the mean is set to find upper and lower limits. This choice of multiplication factor as 3 comes from the fundamentals of normal distribution. As per 68-95-99.7 rule of normal distribution, 99.7 percent of the observations fall within 3 times of standard deviation from the mean [38], [43] . Therefore, the observations outside this limit can be safely termed as 'outliers'.

$$H_{thr} = M_{prev} + 3 * std_{prev} \tag{3.5}$$

$$L_{thr} = M_{prev} - 3 * std_{prev} \tag{3.6}$$

If no mean shift is detected, observations are tested for variance shift. The standard deviation is calculated as the elements are added into the buffer and compared against the upper threshold limits from *limit_array*. The size of the buffer is used to retrieve the appropriate threshold limit from *limit_array*. For example, the present buffer size is 3, std. dev. of this buffer is compared with *limit_array [3]*. As long as the upper threshold limit is not violated, there is no variance shift. If either mean or variance shift is detected, the processing stops and the buffer length value is obtained.

Method - III

The *Method - III* uses the statistical software R (version 3.0.1) to evaluate the buffer length by using inbuilt functions [44]. These inbuilt functions are from package named 'Changepoint' within R [45]. These functions also use mean and variance shift detection to identify the buffer length value. The functions for detecting mean and variance shifts are sophisticated and might be time consuming. It is assumed that the set of measurements follow normal distribution and then a specific function out of all the inbuilt functions is chosen. The function is [46],

multiple.meanvar.norm(data,mul.method="PELT",penalty="SIC",pen.value=0,Q=450, class=TRUE, param.estimates=TRUE) .

In the above defined function, *data* represents the set of measurements and *Q* contains the number of change points of mean and variance to be detected. PELT is name of the algorithm used by this function to identify the change points [47]. The challenge in using this method is feeding the whole set of phasor measurements into R. The next step is to execute the inline functions for mean and variance detection and store the first shift for either mean or variance in an excel sheet. For other two methods, the algorithms are embedded in the SE program to determine the optimal buffer length. In this method, the buffer length values from R are actually fed into SE program through the excel sheet and then the program calculates mean and standard deviation of measurements to proceed further.

3.3 Alternate methods comparison

Variable buffer length numbers from the three different methods can be compared to choose the best method. The following Tables 3.1 -3.6 contain buffer length values along with their mean and standard deviation output for sets 1 and 3 respectively. The values in the given tables are obtained after the pre-processing step. The buffer length values reflect that the result from the three methods used are cloe for some PMUs and not for other PMUs. The presence of both the mean and variance shifts in phase angle measurements is more than that of voltage magnitude measurements. Hence, finding the buffer length for phase angles is more challenging and a close observation shows that in some instances *Method – I* has large buffer length values such as 100 and 790 in Table 3.1; and 840 and 860 in Table 3.4. However, for those instances, the respective buffer length values from the other two methods are relatively smaller and close in size to each other. The reason is due to simplistic nature of the algorithm followed in *Method – I*. The buffer length values are the key to calculate mean and standard deviation of the phasor measurement which in turn would be merged with the conventional measurement set to obtain hybrid SE. The three different methods are employed individually and the respective output estimates are used separately for performance evaluation. More details on performance evaluation between conventional vs. hybrid SE and fixed vs. variable buffer lengths is presented in Chapter 4.

Table 3.1 Buffer length values from the three methods – set 1

PMU	**Voltage magnitudes (p. u.)**			**Relative phase angles (degrees)**		
S. No	***Method - I***	***Method - II***	***Method - III***	***Method - I***	***Method - II***	***Method - III***
1	20	4	10	10	8	10
2	10	2	18	10	8	10
3	20	4	12	10	8	10
4	10	2	12	10	8	10
5	1	6	3	10	8	10
6	10	2	5	10	8	10
7	20	3	12	10	8	10
8	20	4	18	10	8	10
9	10	2	3	15	9	10
10	10	4	12	10	8	10
11	20	3	7	10	8	10
12	10	4	11	10	8	10
13	10	2	10	10	8	10
14	5	2	2	10	8	10
15	10	2	8	100	9	9
16	10	4	4	10	8	10
17	10	2	5	10	9	10
18	5	2	16	10	8	10
19	10	4	18	10	8	10
20	20	3	12	10	8	10
21	20	6	12	10	8	10
22	15	2	3	790	8	3
23	10	2	11	10	8	10
24	20	3	6	10	8	10
25	15	3	3	10	2	10
26	15	2	13	10	8	10
27	10	2	5	10	8	10
28	5	2	2	10	8	10
29	5	4	4	10	8	10
30	5	2	21	10	8	10

Table 3.2 Mean value of measurements from variable buffer length values – set 1

PMU	Voltage magnitudes (p. u.)			Relative phase angles (degrees)		
S. No	***Method - I***	***Method - II***	***Method - III***	***Method - I***	***Method - II***	***Method - III***
1	1.00839	1.00842	1.00841	17.12562	17.12545	17.12562
2	1.02244	1.02243	1.02240	14.55431	14.55415	14.55431
3	1.02248	1.02253	1.02252	16.17096	16.17079	16.17096
4	1.01848	1.01845	1.01847	16.04453	16.04437	16.04453
5	1.03639	1.03633	1.03627	12.27067	12.27050	12.27067
6	1.02131	1.02132	1.02134	15.67319	15.67301	15.67319
7	1.01821	1.01826	1.01824	15.11159	15.11141	15.11159
8	1.02185	1.02185	1.02183	14.85699	14.85681	14.85699
9	1.07570	1.07568	1.07568	14.85334	14.85360	14.85358
10	1.02374	1.02373	1.02374	15.69679	15.69662	15.69679
11	1.07194	1.07197	1.07197	24.07342	24.07325	24.07342
12	1.02231	1.02230	1.02231	16.63429	16.63411	16.63429
13	1.04744	1.04740	1.04744	18.95046	18.95029	18.95046
14	1.02444	1.02444	1.02444	16.56347	16.56330	16.56347
15	1.07289	1.07288	1.07287	5.85864	5.86349	5.86349
16	1.01708	1.01708	1.01708	16.09654	16.09636	16.09654
17	1.04548	1.04551	1.04554	27.26674	27.26663	27.26674
18	1.01269	1.01266	1.01263	18.46651	18.46633	18.46651
19	1.02616	1.02618	1.02612	14.86036	14.86019	14.86036
20	1.04841	1.04841	1.04843	16.73955	16.73937	16.73955
21	1.03261	1.03266	1.03264	19.77773	19.77756	19.77773
22	1.06540	1.06536	1.06536	-1.28662	-1.29583	-1.29582
23	1.01549	1.01545	1.01549	14.89869	14.89854	14.89869
24	1.03699	1.03701	1.03702	14.75779	14.75762	14.75779
25	1.08667	1.08671	1.08671	37.77961	37.78004	37.77961
26	1.04510	1.04515	1.04508	39.92210	39.92200	39.92210
27	1.08600	1.08600	1.08600	21.62859	21.62842	21.62859
28	1.08596	1.08600	1.08600	21.62727	21.62710	21.62727
29	1.08239	1.08241	1.08241	24.27195	24.27179	24.27195
30	1.01443	1.01438	1.01438	20.17896	20.17878	20.17896

Table 3.3 Measurement standard deviation from variable buffer length values – set 1

PMU	**Voltage magnitudes (p. u.)**			**Relative phase angles (degrees)**		
S. No	***Method - I***	***Method - II***	***Method - III***	***Method - I***	***Method - II***	***Method - III***
1	1.50E-05	3.71E-06	8.32E-06	1.79E-04	1.640E-04	1.789E-04
2	1.71E-05	2.61E-05	1.95E-05	1.74E-04	1.661E-04	1.736E-04
3	1.83E-05	1.63E-05	1.28E-05	1.82E-04	1.706E-04	1.821E-04
4	1.08E-05	1.09E-05	1.05E-05	1.79E-04	1.684E-04	1.789E-04
5	1.65E-05	3.97E-05	6.28E-05	1.85E-04	1.735E-04	1.851E-04
6	1.21E-05	4.35E-06	1.34E-05	1.88E-04	1.741E-04	1.884E-04
7	1.74E-05	7.67E-06	1.01E-05	1.86E-04	1.740E-04	1.858E-04
8	2.18E-05	2.23E-05	1.82E-05	1.90E-04	1.784E-04	1.902E-04
9	1.71E-05	9.87E-06	1.26E-05	1.04E-04	3.424E-05	3.784E-05
10	1.36E-05	7.32E-06	1.20E-05	1.88E-04	1.785E-04	1.877E-04
11	1.63E-05	9.17E-06	9.80E-06	1.77E-04	1.672E-04	1.774E-04
12	1.69E-05	2.01E-05	1.63E-05	1.90E-04	1.749E-04	1.895E-04
13	1.98E-05	1.70E-05	1.98E-05	1.84E-04	1.743E-04	1.839E-04
14	1.47E-05	2.17E-06	2.17E-06	1.78E-04	1.685E-04	1.779E-04
15	1.87E-05	9.87E-06	1.81E-05	5.90E-04	9.633E-06	9.633E-06
16	1.27E-05	3.71E-06	3.71E-06	1.88E-04	1.735E-04	1.877E-04
17	2.21E-05	4.00E-06	1.49E-05	1.73E-04	1.577E-04	1.734E-04
18	2.32E-05	1.52E-05	1.62E-05	1.87E-04	1.764E-04	1.871E-04
19	1.43E-05	8.40E-06	1.88E-05	1.87E-04	1.733E-04	1.870E-04
20	1.85E-05	5.46E-06	1.03E-05	1.93E-04	1.826E-04	1.934E-04
21	1.85E-05	2.15E-05	1.32E-05	1.81E-04	1.736E-04	1.810E-04
22	1.54E-05	9.87E-06	1.26E-05	2.28E-04	4.639E-06	8.393E-06
23	1.08E-05	2.17E-06	1.03E-05	1.72E-04	1.647E-04	1.721E-04
24	1.71E-05	8.07E-06	8.36E-06	1.84E-04	1.724E-04	1.839E-04
25	1.83E-05	6.67E-06	6.67E-06	1.37E-04	1.965E-04	1.371E-04
26	7.40E-06	1.16E-05	1.17E-05	1.10E-04	1.090E-04	1.102E-04
27	4.28E-05	9.87E-06	1.26E-05	1.79E-04	1.668E-04	1.792E-04
28	1.82E-05	9.87E-06	1.26E-05	1.79E-04	1.662E-04	1.793E-04
29	2.17E-05	8.87E-06	8.87E-06	1.70E-04	1.579E-04	1.701E-04
30	2.17E-05	6.52E-06	1.65E-05	1.80E-04	1.658E-04	1.803E-04

Table 3.4 Buffer length values from the three methods – set 3

PMU	**Voltage magnitudes (p. u.)**			**Relative phase angles (degrees)**		
S. No	***Method - I***	***Method - II***	***Method - III***	***Method - I***	***Method - II***	***Method - III***
1	45	3	26	10	8	9
2	25	9	9	10	8	9
3	20	2	21	1	8	2
4	25	2	24	10	8	9
5	60	2	5	10	8	9
6	25	3	26	10	8	9
7	25	3	26	10	8	9
8	60	3	12	10	3	9
9	25	3	13	25	2	2
10	25	2	12	10	8	9
11	25	2	10	10	8	9
12	10	2	24	10	8	9
13	20	10	12	10	8	9
14	25	6	10	10	8	9
15	3	3	8	840	8	3
16	25	2	12	10	8	9
17	25	4	15	10	8	9
18	25	5	20	10	2	9
19	45	3	26	10	8	9
20	50	5	29	10	8	9
21	25	4	26	10	8	9
22	15	2	2	885	8	15
23	60	6	29	10	8	9
24	10	5	12	10	8	9
25	10	2	13	10	2	4
26	10	2	11	10	8	9
27	25	2	5	10	8	9
28	10	2	9	10	8	9
29	25	4	4	10	8	10
30	360	2	16	10	8	9

Table 3.5 Mean value of measurements from variable buffer length values – set 3

PMU	**Voltage magnitudes (p. u.)**			**Relative phase angles (degrees)**		
S. No	***Method - I***	***Method - II***	***Method - III***	***Method - I***	***Method - II***	***Method - III***
1	1.02929	1.02918	1.02915	18.56598	18.56604	18.56602
2	1.00859	1.00861	1.00861	20.10779	20.10784	20.10782
3	1.02551	1.02561	1.02551	20.07337	20.08595	20.07702
4	1.02258	1.02258	1.02258	19.52416	19.52422	19.52420
5	1.04453	1.04448	1.04448	16.64437	16.64443	16.64441
6	1.02709	1.02713	1.02708	19.50267	19.50273	19.50271
7	1.02351	1.02352	1.02351	18.82392	18.82396	18.82395
8	1.02876	1.02870	1.02874	18.83622	18.83619	18.83626
9	1.07525	1.07523	1.07521	13.77712	13.77744	13.77744
10	1.02711	1.02710	1.02711	19.07986	19.07993	19.07990
11	1.07273	1.07276	1.07274	27.05189	27.05194	27.05192
12	1.02614	1.02621	1.02612	20.19765	20.19771	20.19768
13	1.05086	1.05089	1.05088	22.37185	22.37190	22.37188
14	1.02824	1.02829	1.02827	20.12651	20.12656	20.12654
15	1.06992	1.06992	1.06988	4.72091	4.71433	4.71432
16	1.02103	1.02106	1.02103	19.58181	19.58187	19.58185
17	1.05578	1.05577	1.05574	32.39644	32.39650	32.39648
18	1.01557	1.01556	1.01555	21.49573	21.49565	21.49576
19	1.03278	1.03267	1.03263	18.82025	18.82031	18.82029
20	1.05068	1.05057	1.05057	20.14627	20.14633	20.14631
21	1.03333	1.03331	1.03333	23.08301	23.08307	23.08304
22	1.06189	1.06200	1.06200	-2.53314	-2.53862	-2.53862
23	1.02166	1.02156	1.02155	18.65960	18.65966	18.65964
24	1.04238	1.04239	1.04238	18.34700	18.34706	18.34704
25	1.08777	1.08780	1.08778	39.56472	39.56461	39.56472
26	1.04714	1.04718	1.04714	41.94835	41.94848	41.94843
27	1.08814	1.08814	1.08814	24.05188	24.05193	24.05191
28	1.08812	1.08814	1.08814	24.05122	24.05127	24.05125
29	1.08324	1.08326	1.08326	27.17609	27.17613	27.17609
30	1.01856	1.01852	1.01855	19.60289	19.60295	19.60293

Table 3.6 Measurement standard deviation from variable buffer length values – set 3

PMU	**Voltage magnitudes (p. u.)**			**Relative phase angles (degrees)**		
S. No	***Method - I***	***Method - II***	***Method - III***	***Method - I***	***Method - II***	***Method - III***
1	3.03E-05	7.25E-06	2.29E-05	5.49E-05	4.02E-05	4.04E-05
2	1.02E-05	8.76E-06	8.76E-06	4.92E-05	4.41E-05	4.26E-05
3	1.17E-05	1.96E-05	1.11E-05	3.67E-04	2.23E-03	3.65E-03
4	1.04E-05	2.17E-06	9.37E-06	5.15E-05	3.98E-05	3.99E-05
5	1.66E-05	1.02E-05	1.11E-05	5.35E-05	3.84E-05	3.97E-05
6	1.57E-05	5.80E-06	1.51E-05	5.94E-05	4.50E-05	4.44E-05
7	7.96E-06	1.47E-05	7.65E-06	4.42E-05	3.54E-05	3.52E-05
8	2.39E-05	1.47E-05	3.09E-05	5.18E-05	5.13E-05	3.54E-05
9	2.12E-05	2.67E-05	1.91E-05	5.79E-05	4.23E-06	4.23E-06
10	1.23E-05	8.70E-06	1.29E-05	5.63E-05	3.69E-05	4.08E-05
11	9.88E-06	1.00E-06	9.54E-06	4.07E-05	3.39E-05	3.41E-05
12	1.69E-05	1.74E-05	1.22E-05	5.54E-05	4.55E-05	4.53E-05
13	1.13E-05	1.22E-05	1.22E-05	4.43E-05	3.78E-05	3.65E-05
14	1.20E-05	1.66E-05	1.29E-05	4.47E-05	3.56E-05	3.53E-05
15	4.62E-05	4.62E-05	3.38E-05	1.20E-04	4.33E-06	7.21E-06
16	1.05E-05	6.52E-06	9.71E-06	4.77E-05	3.42E-05	3.58E-05
17	1.15E-05	8.66E-06	1.06E-05	5.41E-05	4.48E-05	4.47E-05
18	9.87E-06	8.76E-06	7.77E-06	4.30E-05	4.56E-05	3.43E-05
19	3.07E-05	1.65E-05	2.14E-05	5.58E-05	4.23E-05	4.20E-05
20	1.99E-05	1.77E-05	1.05E-05	4.82E-05	3.80E-05	3.85E-05
21	1.17E-05	1.46E-05	1.13E-05	5.44E-05	4.09E-05	4.28E-05
22	2.04E-05	1.02E-05	1.11E-05	1.07E-04	4.24E-06	3.41E-06
23	1.73E-05	8.40E-06	1.15E-05	5.34E-05	4.02E-05	4.17E-05
24	5.72E-06	6.54E-06	5.00E-06	5.50E-05	4.23E-05	4.35E-05
25	9.63E-06	1.02E-05	8.86E-06	7.85E-05	1.70E-05	6.90E-05
26	1.16E-05	1.01E-05	1.11E-05	1.11E-04	7.39E-05	8.24E-05
27	1.36E-16	1.02E-05	1.11E-05	4.96E-05	4.24E-05	4.22E-05
28	2.14E-05	1.02E-05	1.11E-05	5.05E-05	3.98E-05	4.08E-05
29	8.77E-06	5.00E-06	5.00E-06	3.82E-05	3.70E-05	3.82E-05
30	1.21E-05	1.09E-05	1.40E-05	5.68E-05	3.92E-05	4.24E-05

3.4 Pre-processing required for PMU observations

Time skew

The phase angle measurements need pre-processing before use in the algorithms. A generator bus among the PMU buses is chosen as the reference bus and relative phase angles are calculated for every absolute angle measurements. As mentioned in Section 1.5, these relative phase angle measurements may contain time skew and it is mandatory to remove this component which affects the accuracy of the measurements. The program developed in [36] is adopted for this removal of time skew error. Kalman filtering, which is very effective in extracting the actual signal from the measurements, is used in this program. It is a recursive optimal estimator based on the state space representation [48].

Validate PMU measurements

The buffer length algorithm makes the best of phasor measurements that are reliable. However, phasor measurements from certain PMUs could contain errors and might impact the overall performance of the state estimation. Measurements from such PMUs might have to be completely removed from SE or error correction methods can be applied. In this analysis, these phasor measurements are removed from input measurement set. This sort of removal is simpler for relative phase angle measurements rather than voltage magnitude measurements. For the former, SE solution from utility is used. The mean relative phase angle measurements outputted from buffer length algorithms are compared with the relative phase angle estimates from the utility solution provided for SE. The measurements which are more than ± 3 degrees away from that of solution estimates are removed from the phasor measurements set. For voltage magnitudes, such comparison is not obvious and hence it is difficult to ascertain particular measurements as erroneous. Consequently, the erroneous voltage magnitude measurements are removed only through post-processing. After the process of SE including PMU measurements, it is observed that a voltage magnitude measurement from a particular PMU is causing voltage residuals to increase. Here, voltage residuals refers to difference between voltage magnitude measurements and estimates. The particular voltage measurement that causes this problem is consistent across all sets. Hence, it is fair to conclude that this measurement is erroneous due to unknown reasons and thus removed from input voltage measurements. ‘TS – ROS W.Bus’ is the PMU removed from voltage magnitude measurements all together for all the sets. Relative phase angle measurements from ‘SI – CO’ PMU is observed to be more than 3 degrees deviant from solution estimates and removed from input measurements for all the sets. As an exception to this, ‘WD-BD1’ PMU is also found to have deviant phase angle measurements for set 1. Finally, the conventional active and reactive injection measurements are removed from the buses where PMU measurements are available. The latter is replaced with voltage magnitude and relative phase angle measurements from PMUs. This is a mandatory step to realize performance improvement after including phasor measurements into SE. More explanation on the metrics used for this performance evaluation and the results of the metrics in various cases are provided in the Chapter 4.

Chapter 4. A discussion of results and analysis

4.1 Conventional vs. hybrid SE

The benefits of using hybrid SE can be justified only if there is improvement in state estimates in comparison to the traditional SE. This has been established in a number of research papers [49] - [52]. However, these papers dealt with test systems where the true states are available. Hence, it is straightforward to compare the output estimates with the true solution of the states and exhibit improvement after including PMUs. In this research, real time SE is under study and there is no specific (exact) true solution available. However, there is a solution available for this real time SE from the utility, it is through a traditional SE approach using weighted least squares. Hence, it is not fair to compare state estimates from hybrid SE with traditional SE. Instead of using the states directly as metrics, active and reactive power injection residuals are used for this comparison. The reason for using these residuals is that in the given data all buses have active and reactive injection measurements unlike power flow measurements. The SE that gives lower residuals will be judged as being better. Reactive power injections are used to show improvement in voltage magnitude estimates whereas active power injections are used for relative phase angle estimates. The intent is to show improvement in estimates at three levels of neighboring buses to each PMU. Here, the level refers to number of branches between the neighboring bus and the PMU bus as depicted in Figure 4.1.

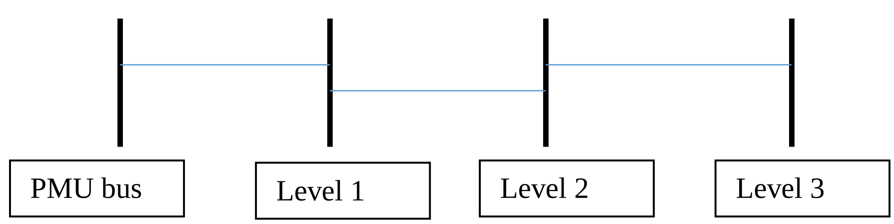

Figure 4.1 Diagram for showing various levels of neighboring buses

Instead of directly using the residuals for comparison, norms of these residuals are used. Various norms used for this analysis are

- 1 – norm
- 2 – norm
- Infinite – norm

$$| X |_p = \sqrt[p]{\sum_{i=1}^{n} | x_i^p |} \tag{4.1}$$

$$| X |_\infty = \max_i | x_i |. \tag{4.2}$$

Of these norms, the 2 - norm is more directly related to WLS method than the other norms. The reason that the 2-norm is commonly used is that the Euclidean distance is the nominal measure of distance in the mathematical sense, and also the derivative of the 2-norm is readily calculated for minimization purposes. The values for 2 - norm at all three levels of neighboring buses for both active and reactive power injection, and voltage magnitude residuals are shown in Figure 4.2 - 4.10. It can be seen from these figures that the improvement in residuals is inversely proportional to the number of levels away from the PMU bus. This is expected because further the neighboring buses are away from the PMU bus, lower the impact of PMU would be.

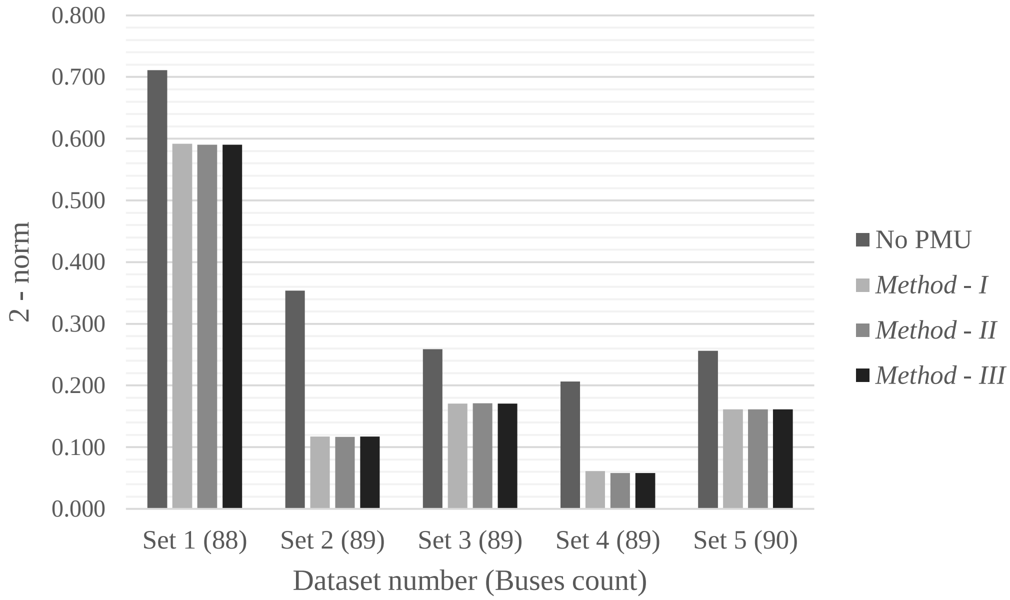

Figure 4.2 Active power injection residuals (p. u.) – Level 1

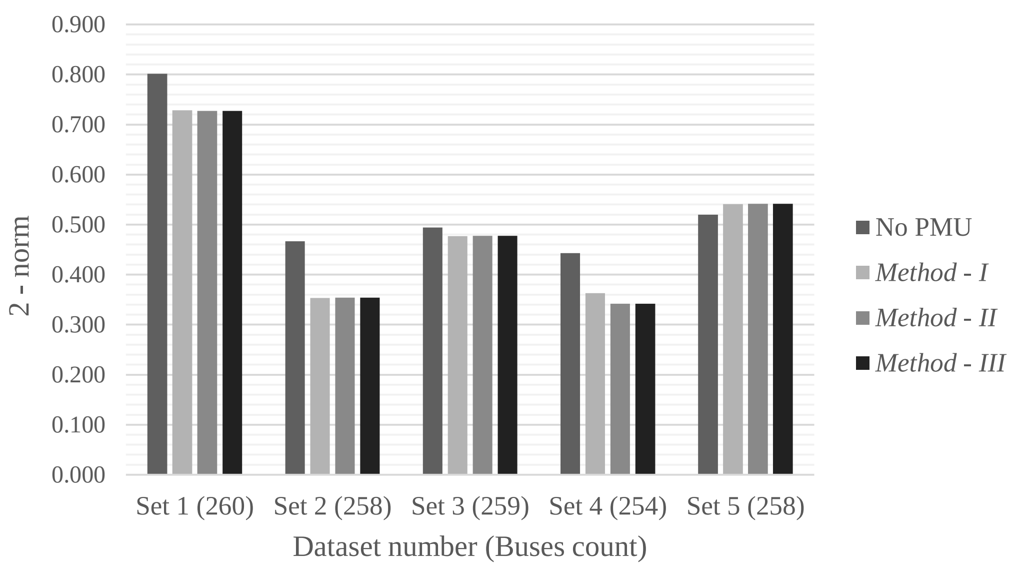

Figure 4.3 Active power injection residuals (p. u.) – Level 2

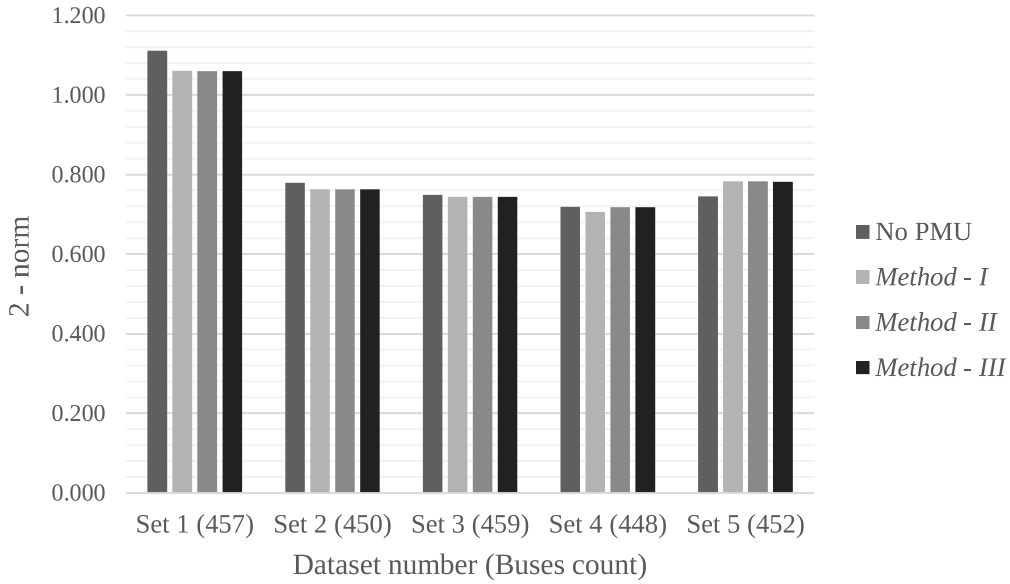

Figure 4.4 Active power injection residuals (p. u.) – Level 3

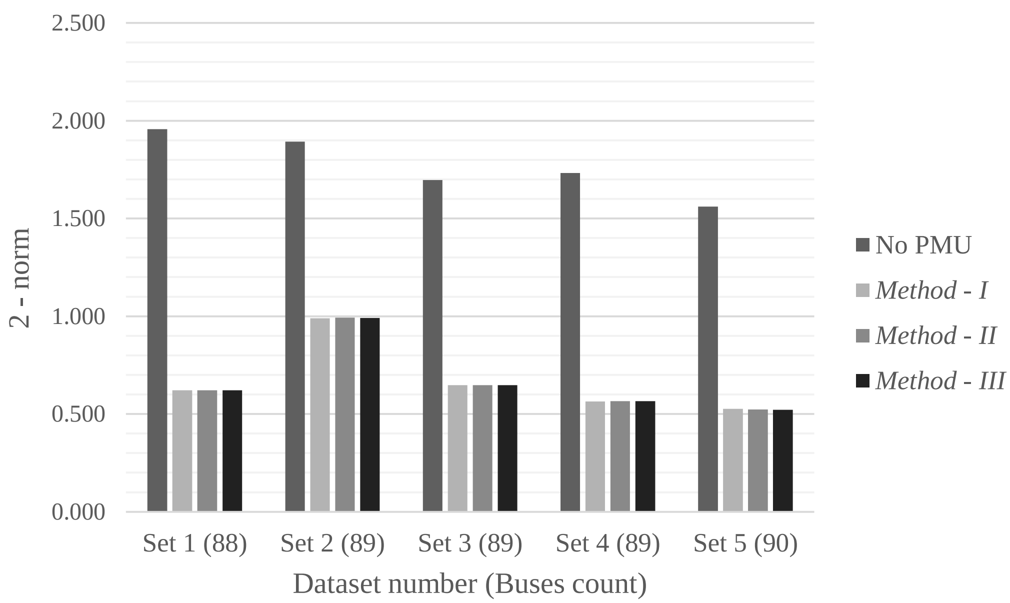

Figure 4.5 Reactive power injection residuals (p. u.) – Level 1

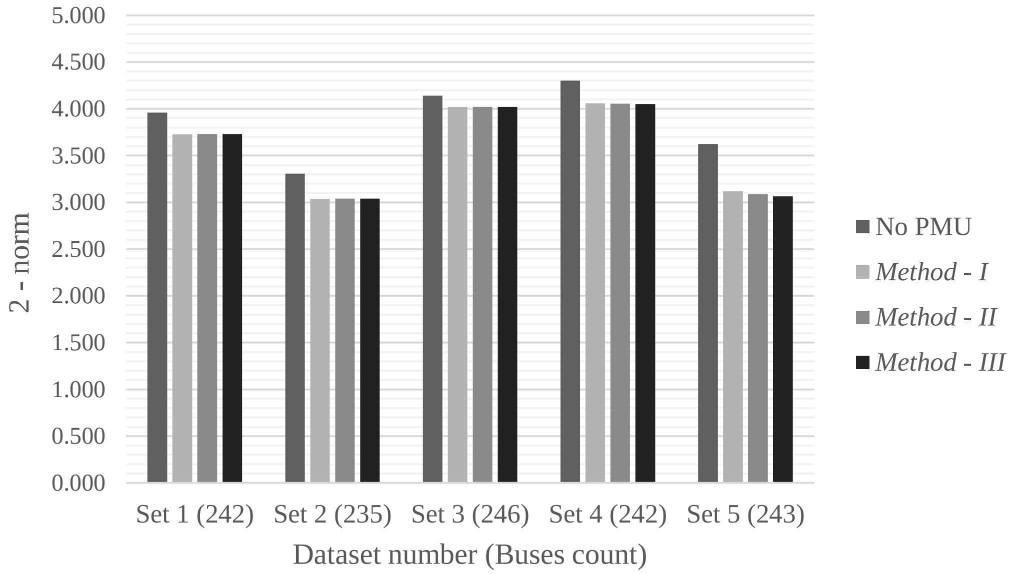

Figure 4.6 Reactive power injection residuals (p. u.) – Level 2

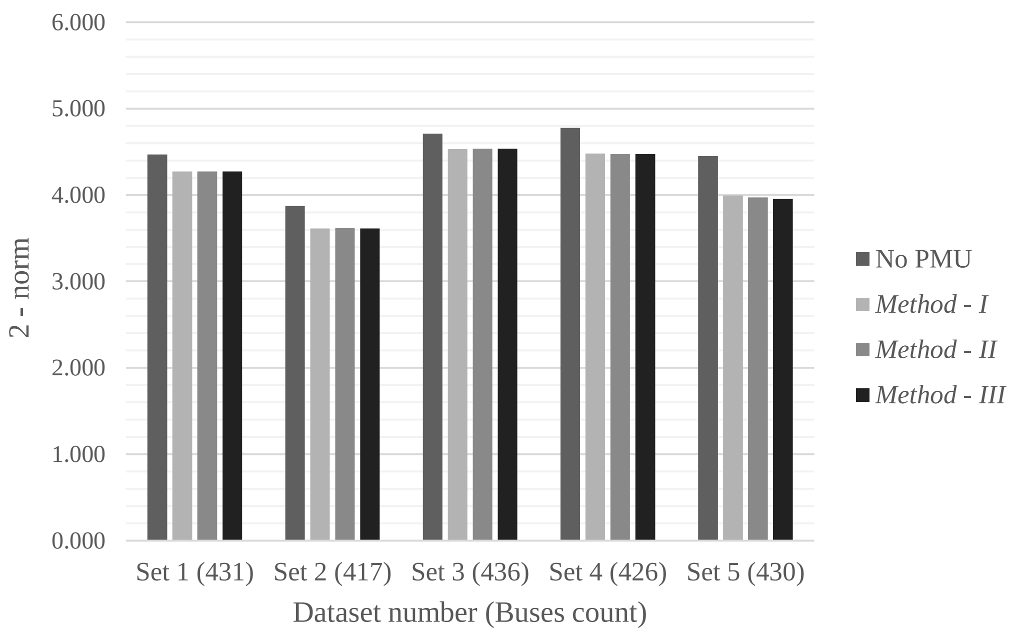

Figure 4.7 Reactive power injection residuals (p. u.) – Level 3

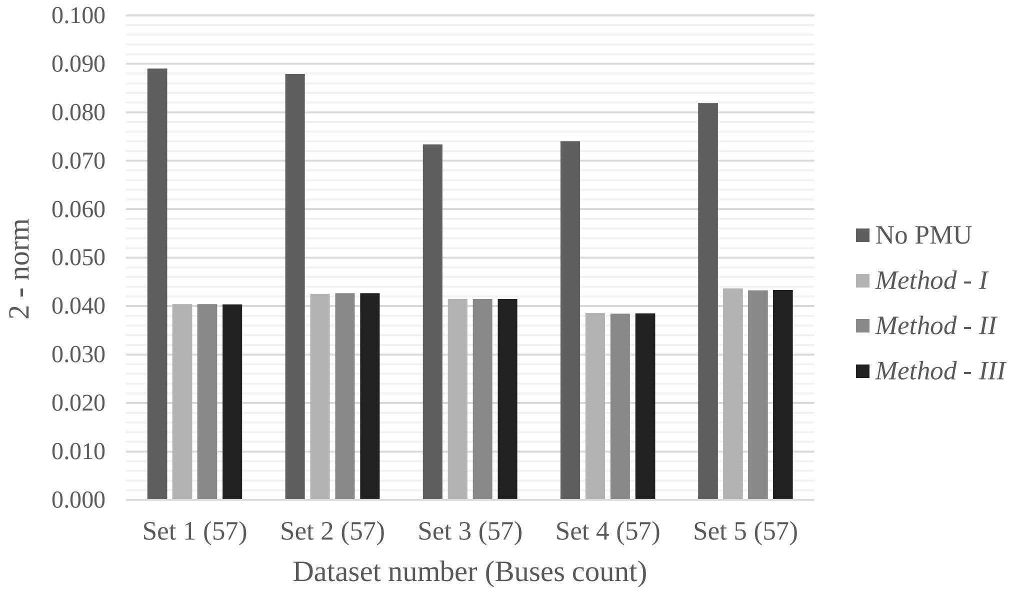

Figure 4.8 Voltage magnitude residuals (p. u.) – Level 1

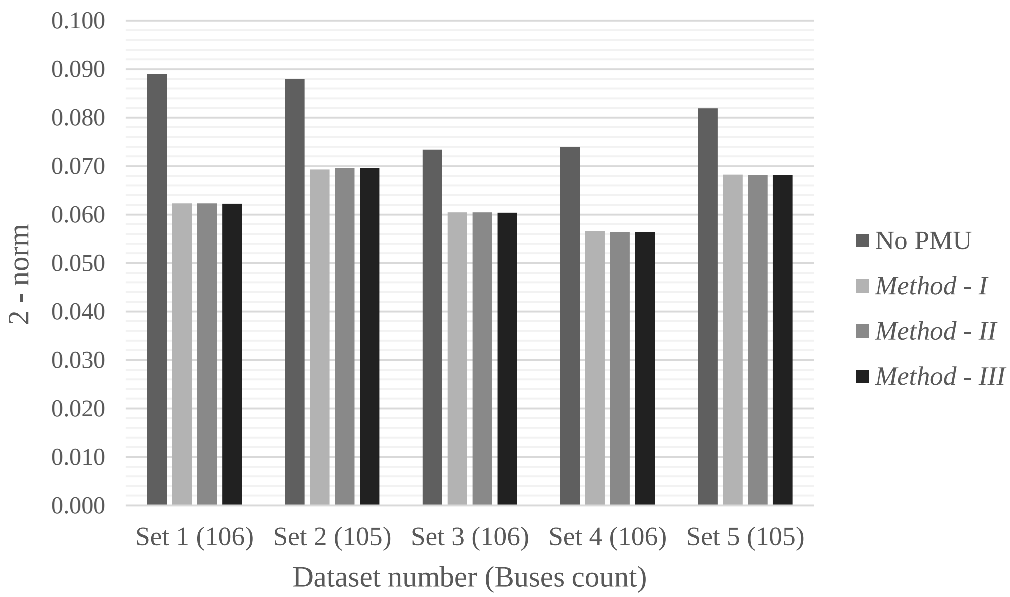

Figure 4.9 Voltage magnitude residuals (p. u.) – Level 2

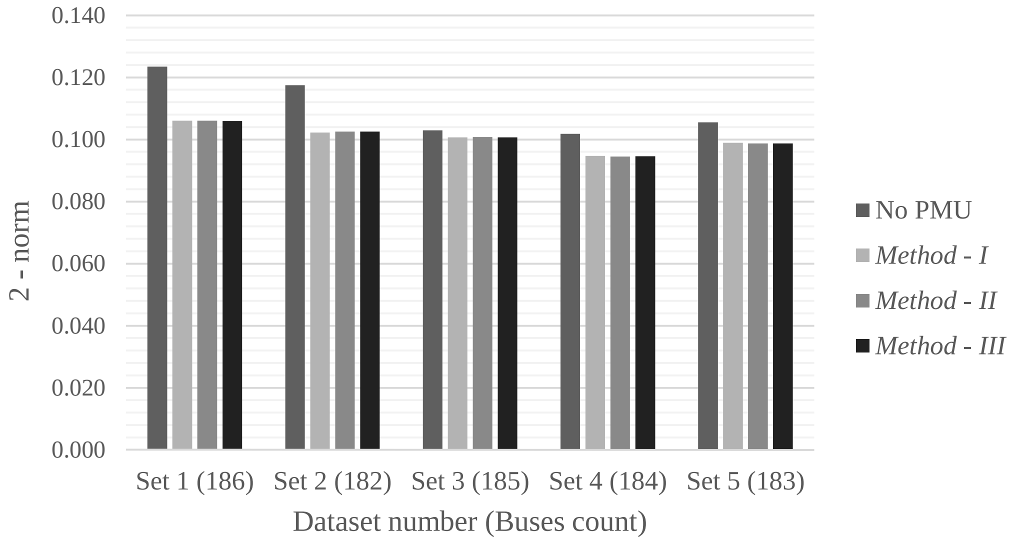

Figure 4.10 Voltage magnitude residuals (p. u.) – Level 3

It can be observed from Figures 4.3 and 4.4 that all the three methods did not improve the residuals in the hybrid SE for level 2 and level 3 neighboring buses. This is due to presence of two PMUs in set 5 data that positively impact the residuals at level 1 neighboring buses but negatively impact the residuals for other levels. After removing the two PMUs (W066WESTWING__01 and W066WESTWING__02), the improvement in residuals is apparent from Figure 4.11 and Figure 4.12. This gives an insight that phasor measurements might not give an expected improvement if they are erroneous. However, this is not observed in the other 4 sets. Hence, this case can be considered special and it is safe to conclude that phasor measurement units improve the majority of SE cases. More detailed comparison involving all the three norms for all the levels can be seen in the table given in Appendix C.

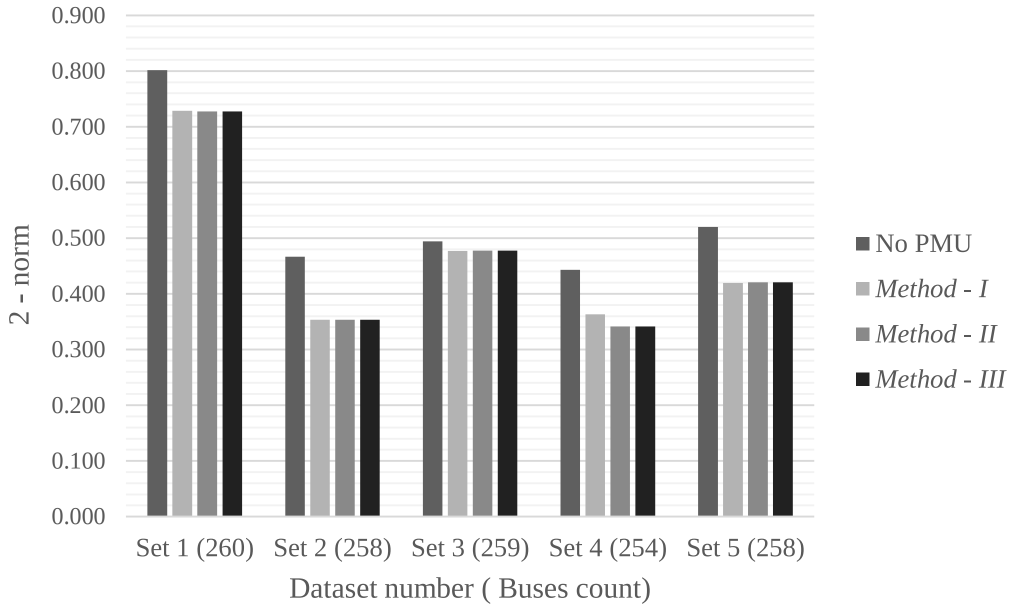

Figure 4.11 Active power injection residuals (p. u.) after removing two PMUs – Level 2

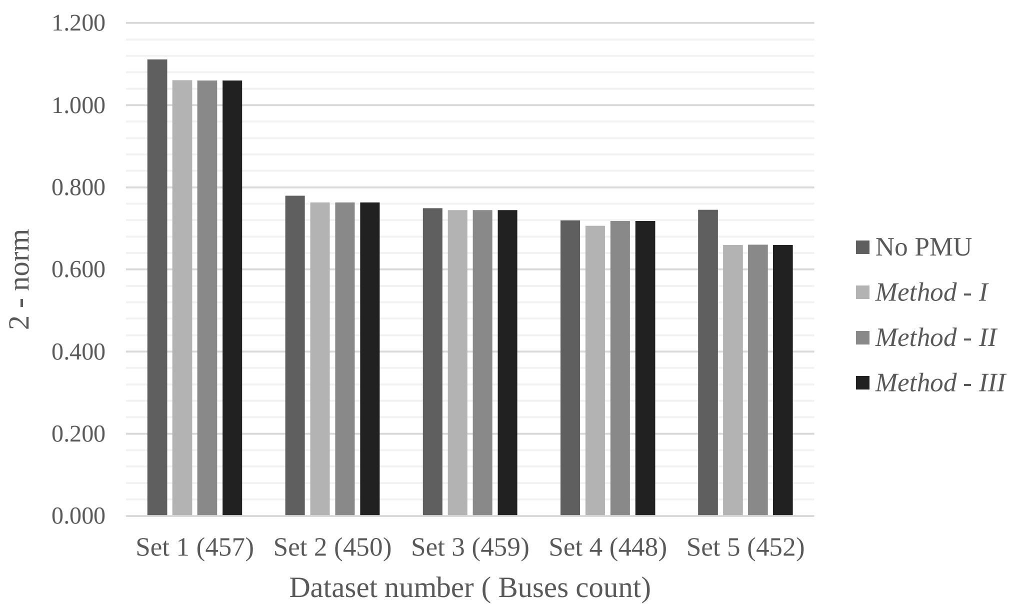

Figure 4.12 Active power injection residuals (p. u.) after removing two PMUs – Level 3

4.2 Fixed vs. variable buffer lengths

The methods for finding optimal buffer lengths are beneficial only if buffer length design exhibits improvement rather than fixing buffer length. Fixed buffer length value refers to assuming one particular buffer length for all the PMUs. The same active and reactive power injection residuals are employed to show improvement with variable over fixed size buffers. Here, the residuals are calculated only at PMU buses because the neighboring buses would get affected by other conventional measurements and hence the difference in residuals between fixed and variable buffer lengths might not be apparent. In future, as the number of PMU installations increase, the performance improvement through usage of variable buffer lengths would also become increasingly apparent. For analysis purposes, the fixed buffer size of 2 for every PMU is compared with variable buffer lengths from the three methods. The size of 2 as fixed buffer length is chosen as opposed to size 1 because the size 1 buffer will result in a standard deviation value of zero. Hence, in order to avoid using any assumed standard deviation value along with buffer size 1, using actual standard deviation from buffer of size 2 is preferred. Comparison between fixed and variable buffer lengths using 2-norm of active and reactive power injection residuals can be seen from Figure 4.13 to Figure 4.14. It can be observed from these figures that using variable buffer length is better than using fixed buffer lengths. Variable buffer length determination using *Method – II* and *Method – III* has shown improvement in all the 5 sets. However, the residuals improvement using *Method – I* can be seen only in 4 sets. The set 4 in Figure 4.13 shows that there is no improvement in residuals using *Method - I.* As explained earlier in section 3.4, this result is due to *Method – I* being less sophisticated than the other two methods.

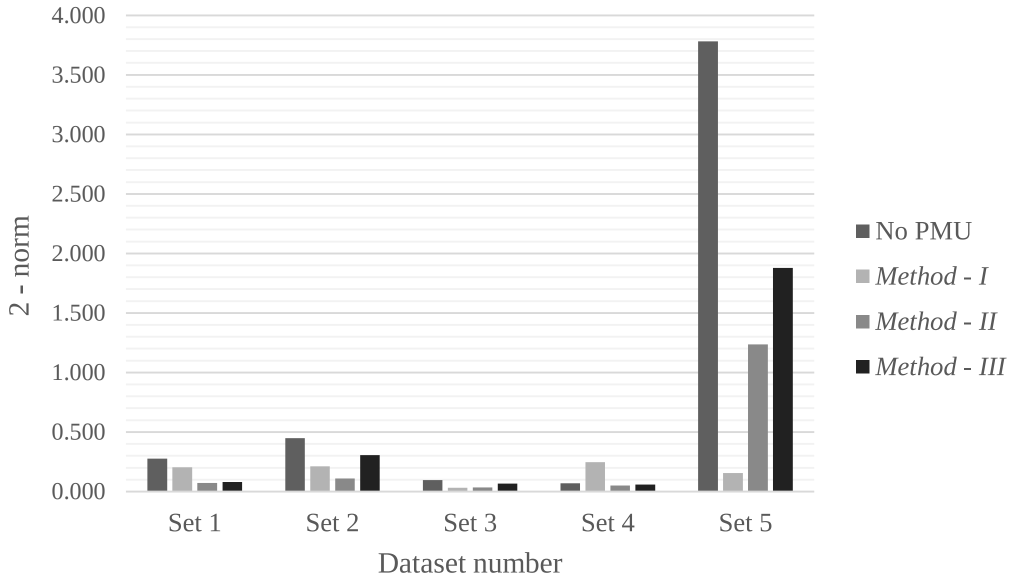

Figure 4.13 Active power injections residuals (p. u.) at PMU buses

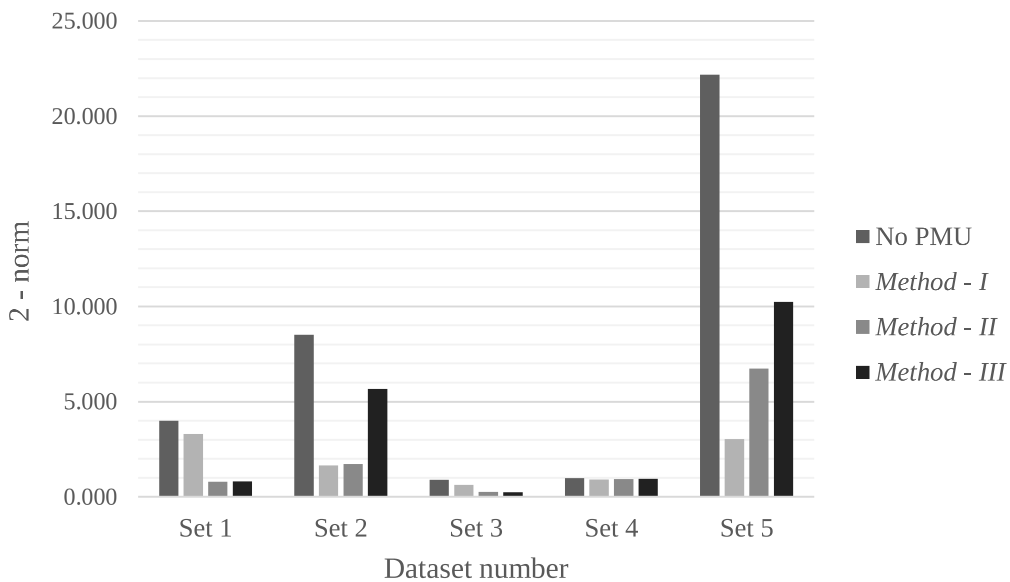

Figure 4.14 Reactive power injections residuals (p. u.) at PMU buses

Table 4.1 Active power injection residuals (p. u.) at PMU buses

Set No.	Norm	BL 2	*Method - I*	*Method - II*	*Method – III*
1	norm 1	0.3926	0.3034	0.1138	0.133
	norm 2	0.2757	0.2051	0.0722	0.081
	norm inf.	0.2633	0.1975	0.00652	0.0692
2	norm 1	0.5675	0.3198	0.171	0.4205
	norm 2	0.4475	0.2131	0.1088	0.3068
	norm inf.	0.4351	0.1998	0.0863	0.2894
3	norm 1	0.2213	0.0461	0.0769	0.121
	norm 2	0.0952	0.0324	0.0343	0.0663
	norm inf.	0.0581	0.0316	0.02258	0.058
4	norm 1	0.1196	**0.7151**	0.0759	0.0934
	norm 2	0.0705	**0.2471**	0.0518	0.06
	norm inf.	0.0495	**0.1152**	0.04791	0.0478
5	norm 1	4.1242	0.2246	1.3094	2.0378
	norm 2	3.7818	0.1566	1.2362	1.8784
	norm inf.	3.7754	0.1523	1.2352	1.8736

Note: Values in **bold** refers to no improvement in residuals

Table 4.2 Reactive power injection residuals (p. u.) at PMU buses

Set No.	Norm	BL 2	Method - I	Method - II	Method - IIII
1	norm 1	4.4553	3.6039	0.9855	1.1405
	norm 2	3.9954	3.292	0.7932	0.8056
	norm inf.	3.9888	3.2896	0.7878	0.7874
2	norm 1	10.1979	3.261	2.4406	6.7156
	norm 2	8.5162	1.6454	1.7186	5.6669
	norm inf.	8.4795	1.2162	1.6791	5.6395
3	norm 1	1.6796	0.8405	0.5172	0.4629
	norm 2	0.8871	0.6177	0.2478	0.2312
	norm inf.	0.7811	0.6013	0.1957	0.1958
4	norm 1	1.5482	1.9236	1.1607	1.3633
	norm 2	0.97	0.9151	0.9339	0.9501
	norm inf.	0.9262	0.666	0.92745	0.9266
5	norm 1	40.2136	5.7103	10.3282	15.60775
	norm 2	22.1723	3.0328	6.737	10.2506
	norm inf.	18.7451	2.6549	6.1061	9.2806

From the detailed comparison involving all the three norms for the PMU buses, it can be seen that in majority of the cases the residuals using Method - II is lower than the other two methods. Method – II is not as sophisticated as Method – III, however, the need for an extra tool such as R software is not present in Method – II. This is an important benefit of using Method – II. The detailed comparison can be seen in Table 4.1 and 4.2.

It can be noted from the Table 4.2 that infinite norm of sets 1, 2 and 5 are high. This shows that a particular PMU has large worst case residuals and consequently increases other norms. This prompts the question whether the improvement in residuals between various methods is only due to this particular PMU. In order to verify this, this particular PMU is removed from the residual calculation and the new residual values are compared. Values of reactive power injection residuals after removing the worst case PMU in respective sets can be seen in Table 4.3. These new residuals reflects that using variable sized buffers for phasor measurements is better than fixed sized buffers.

Table 4.3 Reactive power injection residuals (p. u.) at PMU buses after removing respective deviant PMUs

Set No.	**Norm**	**BL 2**	***Method - I***	***Method - II***	***Method - III***
1	norm 1	0.4418	0.2913	0.1935	0.3484
	norm 2	0.2294	0.1233	0.0924	0.1700
	norm inf.	0.1960	0.0987	0.0772	0.1269
2	norm 1	1.6599	2.0369	0.7503	1.0377
	norm 2	0.7874	1.1082	0.3659	0.5549
	norm inf.	0.5332	0.9782	0.3207	0.4911
5	norm 1	15.0048	4.8505	2.0158	2.8665
	norm 2	8.5847	2.9389	0.9482	1.4691
	norm inf.	7.7257	2.6646	0.5566	1.0182

Chapter 5. Conclusions and future work

5.1 Conclusions

Two key areas relating to SE in power systems are discussed: SE using PMU measurements and buffering the PMU measurements.

Relating to the utilization of PMU measurements, the research presented in this document deals with the idea of proving positive impact with the inclusion of phasor measurements into SE. The problems discussed in this research are difference in reporting rates between PMU and SCADA devices, errors due to random noise in the phasor measurements and variation due to system dynamics.

Prior to analyzing the impact of PMU into SCADA, arriving at state estimates within the specified range from the utility SE solution is achieved. The voltage magnitude estimates and relative phase angle estimates of around 99 percent of the buses within this specified range from utility SE. This is demonstrated in all the five sets. The reason for this step is to prove that benefits of hybrid SE can be realized in the utility state estimator.

Relating to buffering, the concept of using buffered phasor measurements to correct these problems is proposed in [35]. To substantiate this concept, real time SE data and phasor observations from the field provided from a utility are used in this research. Various methods to determine optimal buffer lengths are employed to study the effectiveness of buffered phasor measurements. It can be observed from the presented results that the usage of buffered phasor measurements provides performance enhancement in hybrid SE. This is demonstrated through the residuals as they are lower in the case with a hybrid set of measurements from SCADA and PMU than the case with only conventional measurements from SCADA. This can be observed up to three levels of buses neighboring to all PMU buses. This decrease in residuals is observed more at the buses closer to PMU buses and thereby the improvement in residuals reduces as the level goes higher. This can be clearly seen from Figure 4.2 to 4.4. For example, the largest improvement in active power injection residuals (2- norm) is 66.9 percent for set 2 at level 1 neighboring buses. However, the percentage of improvement for the same residuals and the same set reduces to 24.2 percent at level 2 and further reduces to 2.7 percent at level 3. This is justified because the more the buses are physically away from PMU buses, the impact of PMU would be lesser and the influence of conventional measurements would be more pronounced. From Figure 4.5, the largest percentage improvement for reactive power injection residuals (2- norm) can be seen for set 1 and the value is 68.26 per cent.

Also, the benefits of using variable buffer lengths resulting from the algorithms over using fixed buffer length values is proved. Here, the residual evaluation is performed only at PMU buses where the effect of using variable buffer lengths is apparent. Considering the same data set number (set 2) as an example, it can be seen that improvement in active power injection residuals (2- norm) is 75.6 percent for variable buffer length case with respect to the fixed buffer length case. The corresponding improvement in reactive power injection residuals at PMU buses is 53.35 percent.

It is also observed that the impact of PMU data on state estimation depends on the accuracy of the PMU measurements and does not change with change in load levels. The results shown promote the use of *Method – II* and *Method – III* to arrive at varied sizes of phasor measurement buffer. This would help to get better state estimates at the PMU buses along with the neighboring buses.

5.2 Proposed future work

The key objective of the research focuses on using buffered phasor measurements in SE and analyzing the impact in SE results. In addition to the concept of using buffers, reviewing the optimal placement algorithm for PMUs followed in the utility would bring an additional perspective to this analysis. Analyzing the combined effects of optimal placement and buffer length methods using real time data could be taken for future investigation. Also, including robust bad data removal mechanisms and application of correction factors proposed in [28] could possibly improve the results further.

It would be interesting to experiment the mechanism of automatically accessing R software from the utility state estimator. Achieving this would bring a rigorous determination of optimal buffer length. Although this implementation is beneficial, cost/benefit ratio has to be considered.

In the future, analyzing hybrid SE on a large number of cases, for example 100 cases, would help in capturing the variance of the states. Reduced variance in the states while using PMUs would help in proving performance improvement in hybrid SE. This would be a direct measure of performance. Also, there are additional metrics such as number of critical measurements, percentage of valid solutions and injections errors occurred over time [53]. Developing hundreds of SE cases would help in verifying these new metrics as well. All these metrics together would establish a comprehensive evaluation of performance in hybrid SE.

Testing the effectiveness of the proposed methods in online utility SE could be considered for future investigation. That is, use the buffer length design as described in Chapter 4 in the hardware and software for the PMU data import to the SE. This would help to prove the real time beneficial impacts of hybrid SE and methods proposed for variable buffer length in this research.

REFERENCES

[1] A. Jain and N. R. Shivakumar, “Impact of PMU in dynamic state estimation of power systems,” *North American Power Symp.*, Sept. 2008, pp. 1 - 8.

[2] A. Monticelli, “Electric power system state estimation,” *Proc. of the IEEE*, Feb. 2000, vol. 88, No. 2, pp. 262 - 282.

[3] D. Li, R. Li, Y. Sun, and H. Chen, “State estimation with WAMS/SCADA hybrid measurements,” *IEEE Power and Energy Society General Meeting*, July 2009, pp. 1 - 5.

[4] A. Simoes Costa, A. Albuquerque and D. Bez, “An estimation fusion method for including phasor measurements into power system real-time modeling,” *IEEE Trans. on Power Systems*, May 2013, vol. 28, No. 2, pp. 1910 - 1920.

[5] J. Zhu and A. Abur, “Bad data identification when using phasor measurements,” *Proc. IEEE Power Tech conf., Lausanne*, July 2007, pp. 1676 - 1681.

[6] H. Wu and J.Giri, “PMU impact on state estimation reliability for improved grid security,” *Proc. IEEE Power Energy Soc. Transm. Distrib. Conf. Exhib.*, May 2006, pp. 1349 - 1351.

[7] M. Gol, A. Abur and F. Galvan, “Metrics for success: performance metrics for power system state estimators and measurement designs,” *IEEE Power and Energy Magazine*, Sep. 2012, vol. 10, No. 5, pp. 50 - 57.

[8] A. Ghassemian and B. Fardanesh, “Phasor assisted state estimation for NYS transmission system - implementation and testing,” *Proc. IEEE / PES Power Systems Conf. Expo.*, Mar. 2009, pp. 1 - 8.

[9] A. Monticelli, “Modeling circuit breakers in weighted least squares state estimation,” *IEEE Trans. on Power Systems*, Aug. 1993, vol. 8, No. 3, pp. 1143 - 1149.

[10] E. Castillo, A.J. Conejo, R.E. Pruneda and C.Solares, “Observability analysis in state estimation: A unified numerical approach,” *IEEE Trans. on Power Systems*, May 2006, vol. 21, No. 2, pp. 877 – 886.

[11] I. N. Kolosok, E. S. Korkina, and A. S. Paltsev, “Bad data detection at decomposition of state estimation problem,” *Proc. IEEE Power Tech. Conf., Trondheim*, June 2011, pp. 1 – 6.

[12] A. Abur and A. Gómez-Expósito, *Power system state estimation: theory and implementation*, New York: Marcel Dekker, 2004.

[13] G. N. Korres, “A robust method for equality constrained state estimation,” *IEEE Trans. on Power Systems*, May 2002, vol. 17, no. 2, pp. 305–314.

[14] Anonymous, “Notes on observability analysis,” available at: http://en.wikipedia.org/wiki/Observability

[15] N. V. Nguyen, V. Shin and G. Shevlyakov, “Power system state estimation with fusion method,” *Proc. ICCAE Computer and Automation Eng.*, Feb. 2010, pp. 71–76.

[16] B. Xu, and A. Abur, “State estimation of systems with embedded FACTS devices,” *Proc. IEEE Power Tech. Conf., Bologna,* June 2003.

[17] A. G. Phadke, J. S. Thorp, R. F. Nuqui, and M. Zhou, “Recent developments in state estimation with phasor measurements,” *Proc. Power Systems Conf. and Expo.*, Mar. 2009, pp. 1 – 7.

[18] J. S. Thorp, C. E. Seyler, M. Parashar, and A. G. Phadke, “The large scale electric power system as a distributed continuum”, *IEEE Power Engineering Letters*, January 1998, pp. 49 - 50.

[19] A. G. Phadke and J. S. Thorp, *Synchronized phasor measurements and their applications*, Springer, New York, 2010.

[20] J. Chen, and A. Abur, “Improved bad data processing via strategic placement of PMUs,” *IEEE Power and Energy Society General Meeting*, June 2005, vol. 1, pp. 509 - 513.

[21] J. Chen, and A. Abur, “Enhanced topology error processing via optimal measurement design,” *IEEE Trans. on Power Systems*, Aug 2008, vol. 23, No. 3, pp. 845 – 852.

[22] J. Zhu and A. Abur, “Effect of phasor measurements on the choice of reference bus for state estimation,” *IEEE Power and Energy Society General Meeting*, June 2007, pp. 1 - 5.

[23] L. Zhang and A. Abur, “State estimator tuning for PMU measurements,” *Proc. North American Power Symp.*, Boston, MA, Aug. 2011, pp. 1–4.

[24] L. Zhang and A. Abur, “Impact of tuning on bad data detection of PMU measurements,” *Proc. IEEE Innovative Smart Grid Technologies Conf.*, May 2012, pp. 1 - 5.

[25] W. Lei and X. Li, “Research on PMU/SCADA mixed measurements state estimation algorithm with multi-constraints,” *Proc. IEEE Electrical and Electronics Eng. Conf.*, June 2012, pp. 32 - 35.

[26] M. Asprou and E. Kyriakides, “Enhancement of hybrid state estimation using pseudo flow measurements,” *IEEE Power and Energy Society General Meeting*, July 2011, pp. 1 - 7.

[27] M. Korkali and A. Abur, “Reliable measurement design against loss of PMUs with variable number of channels,” *Proc. North American Power Symp., Starkville, MS*, Oct. 2009, pp. 1 - 5.

[28] Q. Zhang, “Analysis of synchronization and accuracy of synchrophasor measurements,” Ph.D. dissertation, Dept. Elect. Eng., Arizona State Univ., Phoenix, AZ, 2012.

[29] *IEEE Standard for Synchrophasors for Power Systems*, IEEE Std. C37.118–2011, Dec. 2011.

[30] “Notes on reporting rate of PMU and SCADA,” available at:
https://www.naspi.org/

[31] Z. Huang, B. Kasztenny, et al. “Performance evaluation of phasor measurement systems,” *IEEE Power and Energy Society General Meeting*, July 2008, pp. 1 – 7.

[32] D. Shi, D. Tylavsky and N. Logic, “An adaptive method for detection and correction of errors in PMU measurements,” *IEEE Trans. on Smart Grid*, Dec. 2012, vol. 3, no. 4, pp. 1575 - 1583.

[33] Q. Zhang, V. Vittal, G. Heydt, N. Logic, S. Sturgill, “The integrated calibration of synchronized phasor measurement data in power transmission systems,” *IEEE Trans. on Power Delivery*, Oct. 2011, vol. 26, no. 4, pp. 2573 – 2581.

[34] D. Georgakopoulos and W.Q. Yang, “Circuit noise reduction by analogue lowpass filtering and data averaging,” *IEEE Electronics Letters*, Sep. 2001, vol. 37, No. 19, pp. 1147 – 1148.

[35] E. B. Loewenstein, “Reducing the effects of noise in a data acquisition system by averaging,” available at:
http://it360.tw/document/an/pdf/an152.pdf

[36] Q. Zhang, Y. Chakhchoukh, et al. “Impact of PMU buffer length on state estimation and its optimization,” *IEEE Trans. on Power Systems*, May 2013, vol. 28, No. 2, pp. 1657 – 1665.

[37] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, “MATPOWER: steady-state operations, planning and analysis tools for power systems research and education," *IEEE Trans. on Power Systems*, vol. 26, no. 1, pp. 12 - 19, Feb. 2011.

[38] A. Wood and B. F. Wollenberg, *Power generation, operation, and control*, New York: Wiley - Interscience, 1996.

[39] A. G. Exposito and A. Abur, “Generalized observability analysis and measurement classification,” *IEEE Trans. on Power Systems*, Aug. 1998, vol. 13, no. 3, pp. 1090–1095.

[40] K. S. Shanmugan, A. M. Breipohl, *Random signals: detection, estimation and data analysis*, New York: Wiley, 1988.

[41] “Description of inbuilt function ‘ttest2’ in MATLAB,” available at:

http://www.mathworks.com/help/stats/ttest2.html

[42] S. Ahn and J. A. Fessler, "Standard errors of mean, variance and standard deviation estimators," available at:
http://web.eecs.umich.edu/~fessler/papers/files/tr/stderr.pdf

[43] "Notes on normal distribution and empirical rule," available at:
http://www.oswego.edu/~srp/stats/6895997.htm

[44] "Version 3.0.1 of statistical software R," available at:
http://www.r-project.org/

[45] "Changepoint package for mean and variance shift detection," available at:
http://cran.r-project.org/web/packages/changepoint/index.html

[46] "Inbuilt functions used in R," available at:
http://cran.r-project.org/web/packages/changepoint/changepoint.pdf

[47] R. Killick, P. Fearnhead, and I.A. Eckley, "Optimal detection of changepoints with a linear computational cost," available at:
http://arxiv.org/pdf/1101.1438.pdf

[48] P. Zarchan and H. Musoff, *Fundamentals of Kalman filtering: a practical approach*, American Institute of Aeronautics & Astronautics, 2009.

[49] X. Qin, T. Bi, and Q. Yang, "Hybrid non-linear state estimation with voltage phasor measurements," *IEEE Power and Energy Society General Meeting*, June 2007, pp. 1 - 6.

[50] F. Wang, W. Zhang, and P. Li, "Optimal incremental placement of PMUs for power system observability," *IEEE Power and Energy Society General Meeting*, July 2012, pp. 1 - 7.

[51] G. D'Antona and M. Davoudi, "Effect of phasor measurement unit on power state estimation considering parameters uncertainty," *Proc. IEEE Int. Workshop AMPS*, Sep. 2012, pp. 48 - 52.

[52] S. Soni, S. Bhil, et al. "Linear state estimation model using phasor measurement unit (PMU) technology," *Proc. IEEE Int. Conf. on CCE*, Sep. 2012, pp. 1 - 6.

[53] "Formats for PTI power flow data," available at:
http://www.ee.washington.edu/research/pstca/formats/pti.txt

[54] R. Avila-Rosales, M. J. Rice, J. Giri, L. Beard, and F. Galvan, "Recent experience with a hybrid SCADA/PMU on-line state estimator," *IEEE Power and Energy Society General Meeting*, July 2009, pp. 1 - 8.

APPENDIX A

DESCRIPTION OF REAL TIME DATA SAMPLE

A.1 Format for input system data

Branch

An example of branch data from the input system data can be seen as shown below Table A.1.

Table A.1 Format for branch data

Br1	Br2	Br3	Br4	Br5	Br6	Br7	Br8	Br9	Br10	Br11
120	127	0.003	0.121	0	188.33	198.8	209.27	0.975	0	1

Br1 - From bus number

Br2 - To bus number

Br3 – Resistance (*R*) (p. u.)

Br4 – Reactance (*X*) (p. u.)

Br5 – Total line charging susceptance (p. u.) (*S*) (p. u.)

Br6 - MVA rating A (long term rating)

Br7 - MVA rating B (short term rating)

Br8 - MVA rating C (emergency rating)

Br9 - Transformer off nominal turns ratio

Br10 - Transformer phase shift angle (degrees)

Br11 - Branch status.

Except the fields Br9 and Br10, the rest of the fields are available in the branch data itself. These two fields are present in transformer data with reference to branch number. This reference branch number is used to place the values of tap ratio and phase angle shift at the appropriate branches.

Bus

Format of the bus data from the input can be seen in Table A.2 as follows

Table A.2 Format for bus data

B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11
259	1	22.462	0.373	0	0	1	1.017	0.182	69	1

B1 - Bus number

B2 - Bus type (Load bus/ generator bus/ reference bus/ isolated bus)

B3 - Active power demand (P_L) (MW)

B4 - Reactive power demand (Q_L) (MVAr)

B5 - Shunt conductance (G_s) (MW demanded at V = 1.0 p.u.)

B6 - Shunt susceptance (B_s) (MVAr injected at V = 1.0 p.u.)

B7 - Area number (positive integer)

B8 - Voltage magnitude (V_m) (p. u.)

B9 - Relative phase angle (V_a) (degrees)

B10 - Base voltage (kV)

B11 - Zone number (positive integer).

All the fields defined above are available in the bus data received. The relative voltage phase angles are calculated from the absolute phase angles. The reference bus needed for this is chosen from the PMU buses for consistency between conventional and hybrid SE. There is only one generator bus among PMU buses and the same is chosen as reference.

Generator

Generator data format for the input is shown below in the Table A.3

Table A.3 Generator data format

G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
396	-1.132	0.486	162	-150	0.9545	100	1	457.00	-5

G1 - Bus number for the generator

G2 - Active power output (P_G) (MW)

G3 - Reactive power output (Q_G) (MVAr)

G4 - Maximum reactive power output (Q_{max}) (MVAr)

G5 - Minimum reactive power output (Q_{min}) (MVAr)

G6 - Voltage magnitude set point ($V_{scheduled}$) (p. u.)

G7 - Total MVA base of this machine (MVA)

G8 – Generator status of operation

G9 - Maximum active power output (P_{max}) (MW)

G10 - Minimum active power output (P_{min}) (MW).

All the above defined formats for system data can be seen in [54].

A.2 Example input measurement format

All types of input measurements have three parts. They are index of the measurement, value of the measurement and MSD. The first part refers to internal bus number for measurements such as power injection, voltage magnitude and voltage phasor angle or refers to the position of branch in the branch input data if the measurement is a power flow measurement.

Table A.4 Example format for power flow and power injection measurements

Index	Mag. (MW)	std. dev. (MW)
110	487.8	5

Index - Branch index for power flow or bus index for power injection

Mag. - Power flow or injection (MW for active or MVAr for reactive)

std. dev. - Measurement standard deviation (in terms of MW or MVAr)

Table A.5 Example format for voltage magnitude and phase angle measurements

Index	Mag. (p. u.)	std. dev. (p. u.)
85	1.026067	0.004928

Index - Bus index (internal bus number)

Mag. - Voltage magnitude or phase angle (p. u. or degrees respectively)

std. dev. - Measurement standard deviation (in terms of p. u. for voltage magnitude or in degrees for relative phase angle).

APPENDIX B

MATLAB CODES DEVELOPED IN THE PROJECT

B.1 Finding the final set of buses

```
% Program to find the buses from the final set of branches
mpc = loadcase('SEInput1set.m');


% 'From' and 'To' bus numbers whose branch connection has
status 2 %
b = [

3     4 ....... 2249

]';


% Get only the unique bus numbers
b = unique(b);


% Initiate variables
disconnect_bus = []; count = 1;


% 'From' bus and 'To' bus numbers of only connected branches %
br_from = mpc.branch(:,1);

br_to = mpc.branch(:,2);


% Loop for finding the buses which are not part of connected
branches %
for i = 1:size(b,1)

      k = b(i);

      fr = find(br_from == k);

      to = find(br_to == k);

      if isempty(fr) && isempty(to)

      disconnect_bus(count,1) = k;

      count = count+1;

end
```

```
end


% Get only unique bus numbers of disconnected buses
disconnect_bus = unique(disconnect_bus);


% All the buses and generators that are currently in MATPOWER
CASE.This set include buses from open ended branches.%
buses = mpc.bus;

gens = mpc.gen;


% All the open ended buses in the present set of buses are re-
moved%
for i = 1:size(disconnect_bus,1)

xx = find(buses(:,1) == disconnect_bus(i));

if ~isempty(xx)

buses(xx, :) = [];

end

end

display('END');
```

B.2 Fetching the required phasor measurements from the data files

```
% List of all PMU names in excel sheet
[PMUNAMES,TXT] = xlsread('PMU and Bus Names1.xlsx','Sheet1');


% Define all the variables
start = 1; endu = 31; limit = 899;

DateTag = '_2013-04-24T134700_3600m'; LoadLevel = 'HighShoulder-
';

% Index of PMU measurement at the instant of SE
SE_start = 109801;
```

```
% Retrieve all data files names in a variable
filenames = TXT(start:endu,3);


% Time stamp to ensure that present instant of SE matches
% with the 900th measurement to be fetched
time_stamp = '2013-04-24T14:48:00.000-07:00';

for i = 1:size(filenames,1)

    filename = cellstr(filenames(i));

    dATE = cellstr(DateTag);

    % Formation of whole filename from different variable
    filename = [LoadLevel filename{1} dATE{1} '.csv'];

    % Reading all contents of data file
    [PMUinfo, Text] = xlsread(filename);



      % Condition to check the present instant vs time stamp in
      the data%
    if strcmp(time_stamp, Text(SE_start+1,1))

           % Fetching only 900 measurements from the present SE
           instant%

        PMU.VM(1:900,i) = PMUinfo((SE_start - limit):SE_start,1);

        PMU.VA(1:900,i) = PMUinfo((SE_start - limit):SE_start,2);

        status(1:900,i) = Text((SE_start - limit):SE_start,2);

    else

      % Return if there is no matching between time stamp & pre-
      sent SE instant %

        display(filename);

        return;

    end

end
```

```
display('END')
```

B.3 Program for implementing *Method – II*

```
% Function definition of Method II
%     Input arguments - 900 Voltage magnitude or phase angle
%     measurements for all PMUs
%     Output arguments - Buffer length, mean value of
%     measurment and its standard deviation for all PMUs
function [bufferlength,buffer_mean,buffer_std] = MethodII(Data)

% Extracting 1st and 2nd dimensions of Data in a variable
max_limit = size(Data,2);

sizeofPMUdata = size(Data,1);



%Calculate initial standard deviation (init_std)
initial_std = [];

for p = 1:max_limit
    for q = (sizeofPMUdata-1):-1:1

        st = std(Data(q:900,p));

        if st ~= 0

            initial_std(p,1) = st;

            break;

        end

    end

end



for it = 1:max_limit

    data = Data(:,it);

    for n = 2:sizeofPMUdata



       % Define variables to use eqn. (3.2)
```

```
     t = (n-1)/2;

     h1 = gamma(n/2);

     h1 = h1*h1;

     h2 = gamma((n-1)/2);

     h2 = h2*h2;

    % Retrieve initial standard deviation
     % only for first time
     if n == 2

         sigma  = initial_std(it,1); end

     % calculate Vn using eqn. (3.2)
     if n < 193

         Vn = 2*(t -(h1/h2));

         xx = sigma*sqrt(Vn/(n-1));

     else

         xx = sigma/sqrt(2*(n-1));

    end

   % calculate threshold limit using eqn. (3.3)
    upplimit(n,1) = sigma + 3*xx;

    sigma = std(data(900-n+1:900));

 end


% Initiate buffer length value to 1
blength = 1;

meanshift = 0;

 for i = size(data,1):-1:1

     if i < sizeofPMUdata


        %Mean, std. dev. and variance estimation for present
        buffer%
```

```
buffer2 = data(i:sizeofPMUdata);

mean_buff2 = mean(buffer2);

bufferstd2 = std(buffer2);

len2 = length(buffer2);


% Mean, std. dev. and variance estimation for previous
buffer %
buffer1 = data(i+1:sizeofPMUdata);

mean_buff1 = mean(buffer1);

std_buff1  = std(buffer1);

len1 = length(buffer1);


% Finding upper and lower limits of mean shift
uppthr = mean_buff1+3*std_buff1/sqrt(len1);

lowthr = mean_buff1-3*std_buff1/sqrt(len1);



% Checking for mean shift
if (len2 > 2) && ((data(i)> uppthr) || (data(i) <
lowthr))

    meanshift = 1;
end


% Checking for variance shift
if (meanshift == 0) && (bufferstd2 < upplimit(len2))

     %No shift detection and hence buffer length I
is  updated%

    blength = len2;

 else

     %Shift detected. Exit for present PMU.
     break;

end
```

```
        end

    end

% Final buffer length for present PMU
bufferlength(it,1) = blength;

% Measurement mean calculated through buffer length
buffer_mean(it,1)  = mean(data(sizeofPMUdata:-1:(sizeofPMUdata-
blength)+1));

% std. dev. calculation through buffer length
buffer_std(it,1)   = std(data(sizeofPMUdata:-1:(sizeofPMUdata-
blength)+1));

end

display('END');
```

APPENDIX C

DETAILED COMPARISON TABLES

The following tables contain residual values for various norms compared between conventional SE and the three methods in hybrid SE.

Table C.1 Residual values for set 1 (p. u.) – Level 1

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	1.6491	0.8551	0.8533	0.8532
	norm 2	0.7113	0.5917	0.5904	0.5904
	norm inf.	0.6142	0.5889	0.5876	0.5876
Reactive power injection	norm 1	6.9597	1.7974	1.7977	1.7968
	norm 2	1.9575	0.6216	0.6217	0.6213
	norm inf.	0.9528	0.3696	0.3699	0.3695
Voltage	norm 1	0.4623	0.2053	0.2051	0.2050
	norm 2	0.0751	0.0404	0.0404	0.0404
	norm inf.	0.0237	0.0131	0.0131	0.0131

Table C.2 Residual values for set 2 (p. u.) – Level 1

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	0.9429	0.3664	0.3662	0.3665
	norm 2	0.3539	0.1173	0.1170	0.1171
	norm inf.	0.2926	0.0995	0.0991	0.0992
Reactive power injection	norm 1	6.0607	2.8285	2.8339	2.8331
	norm 2	1.8937	0.9893	0.9923	0.9916
	norm inf.	1.0815	0.6849	0.6884	0.6875
Voltage	norm 1	0.3673	0.2085	0.2092	0.2089
	norm 2	0.0623	0.0426	0.0427	0.0426
	norm inf.	0.0215	0.0187	0.0187	0.0188

Table C.3 Residual values for set 3 (p. u.) – Level 1

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	0.8826	0.4219	0.4229	0.4218
	norm 2	0.2586	0.1709	0.1710	0.1709
	norm inf.	0.1539	0.1621	0.1621	0.1621
Reactive power injection	norm 1	6.2856	1.8349	1.8357	1.8336
	norm 2	1.6974	0.6474	0.6477	0.6469
	norm inf.	0.8597	0.3750	0.3753	0.3750
Voltage	norm 1	0.3686	0.2056	0.2055	0.2053
	norm 2	0.0608	0.0415	0.0415	0.0414
	norm inf.	0.0215	0.0159	0.0159	0.0159

Table C.4 Residual values for set 4 (p. u.) – Level 1

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	0.7515	0.2838	0.2744	0.2744
	norm 2	0.2063	0.0612	0.0583	0.0583
	norm inf.	0.1296	0.0301	0.0285	0.0285
Reactive power injection	norm 1	6.3126	1.5521	1.5589	1.5596
	norm 2	1.7323	0.5634	0.5650	0.5654
	norm inf.	0.8654	0.3815	0.3803	0.3804
Voltage	norm 1	0.3623	0.1918	0.1909	0.1913
	norm 2	0.0614	0.0386	0.0385	0.0385
	norm inf.	0.0234	0.0129	0.0129	0.0129

Table C.5 Residual values for set 5 (p. u.) – Level 1

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	0.7061	0.4743	0.4743	0.4741
	norm 2	0.2563	0.1613	0.1613	0.1613
	norm inf.	0.2193	0.1468	0.1468	0.1468
Reactive power injection	norm 1	5.9316	1.5534	1.5370	1.5293
	norm 2	1.5611	0.5258	0.5226	0.5218
	norm inf.	0.861	0.3754	0.3754	0.3756
Voltage	norm 1	0.3472	0.2021	0.2001	0.2010
	norm 2	0.0584	0.0436	0.0433	0.0434
	norm inf.	0.0265	0.0230	0.0231	0.0231

Table C.6 Residual values for set 1 (p. u.) – Level 2

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	3.1720	2.7041	2.7004	2.7002
	norm 2	0.8013	0.7287	0.7275	0.7275
	norm inf.	0.6142	0.5889	0.5876	0.5876
Reactive power injection	norm 1	20.3251	14.8808	14.8904	14.8855
	norm 2	3.9585	3.7286	3.7298	3.7290
	norm inf.	2.4580	2.6650	2.6637	2.6634
Voltage	norm 1	0.6908	0.4823	0.4821	0.4816
	norm 2	0.0890	0.0623	0.0623	0.0623
	norm inf.	0.0252	0.0142	0.0142	0.0142

Table C.7 Residual values for set 2 (p. u.)– Level 2

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	2.2981	1.8852	1.8866	1.8863
	norm 2	0.4668	0.3537	0.3537	0.3537
	norm inf.	0.2926	0.1833	0.1833	0.1833
Reactive power injection	norm 1	17.3685	14.7333	14.7601	14.7590
	norm 2	3.3071	3.0367	3.0405	3.0389
	norm inf.	1.2951	1.5094	1.5057	1.4988
Voltage	norm 1	0.6703	0.5035	0.5059	0.5049
	norm 2	0.0879	0.0693	0.0696	0.0696
	norm inf.	0.0322	0.0241	0.0242	0.0242

Table C.8 Residual values for set 3 (p. u.) – Level 2

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	2.4655	2.0745	2.0782	2.0769
	norm 2	0.4941	0.4770	0.4777	0.4776
	norm inf.	0.2724	0.2866	0.2867	0.2865
Reactive power injection	norm 1	21.6243	16.3392	16.3470	16.3423
	norm 2	4.1395	4.0194	4.0222	4.0218
	norm inf.	2.5987	2.6979	2.7009	2.7006
Voltage	norm 1	0.5772	0.4607	0.4607	0.4602
	norm 2	0.0734	0.0605	0.0605	0.0604
	norm inf.	0.0219	0.0159	0.0159	0.0159

Table C.9 Residual values for set 4 (p. u.) – Level 2

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	2.2808	1.8148	1.7458	1.7460
	norm 2	0.4429	0.3632	0.3416	0.3415
	norm inf.	0.2331	0.1753	0.1753	0.1753
Reactive power injection	norm 1	21.1814	16.0454	16.0259	16.0256
	norm 2	4.3025	4.0597	4.0541	4.0530
	norm inf.	2.8435	2.6206	2.6210	2.6195
Voltage	norm 1	0.5747	0.4295	0.4268	0.4275
	norm 2	0.074	0.0566	0.0564	0.0564
	norm inf.	0.0234	0.0129	0.0129	0.0129

Table C.10 Residual values for set 5 (p. u.) – Level 2

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	2.1213	**2.6019**	**2.6078**	**2.6063**
	norm 2	0.52	**0.5408**	**0.5416**	**0.5414**
	norm inf.	0.3681	**0.3096**	**0.3094**	**0.3092**
Reactive power injection	norm 1	20.3126	14.5369	14.3695	14.2550
	norm 2	3.6246	3.1194	3.0878	3.0636
	norm inf.	1.7777	1.4078	1.3491	1.3166
Voltage	norm 1	0.6018	0.4649	0.4613	0.4625
	norm 2	0.0819	0.0683	0.0682	0.0682
	norm inf.	0.0402	0.0302	0.0310	0.0307

Note: Values in bold refers to no improvement in residuals

Table C.11 Residual values for set 1 (p. u.) – Level 3

Residual type	Norm	No PMU	*Method - I*	*Method - II*	*Method - III*
Active power injection	norm 1	6.0274	5.6382	5.6335	5.6334
	norm 2	1.1114	1.0609	1.0595	1.0595
	norm inf.	0.6142	0.5889	0.5876	0.5876
Reactive power injection	norm 1	31.3407	25.9787	25.9858	25.9779
	norm 2	4.4683	4.2725	4.2734	4.2726
	norm inf.	2.4580	2.6650	2.6637	2.6634
Voltage	norm 1	1.1579	1.0194	1.0194	1.0186
	norm 2	0.1235	0.1060	0.1060	0.1060
	norm inf.	0.0396	0.0344	0.0343	0.0343

Table C.12 Residual values for set 2 (p. u.) – Level 3

Residual type	Norm	No PMU	*Method - I*	*Method - II*	*Method - III*
Active power injection	norm 1	4.5599	4.5035	4.5056	4.5062
	norm 2	0.7793	0.7627	0.7629	0.7632
	norm inf.	0.4486	0.4527	0.4527	0.4527
Reactive power injection	norm 1	28.1317	25.1487	25.1773	25.1726
	norm 2	3.8736	3.6130	3.6163	3.6148
	norm inf.	1.2951	1.5094	1.5057	1.4988
Voltage	norm 1	1.1371	0.9621	0.9657	0.9642
	norm 2	0.1175	0.1022	0.1025	0.1025
	norm inf.	0.0356	0.0321	0.0321	0.0322

Table C.13 Residual values for set 3 (p. u.) – Level 3

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	5.0001	4.6901	4.6961	4.6916
	norm 2	0.7489	0.7441	0.7445	0.7444
	norm inf.	0.2769	0.2873	0.2875	0.2871
Reactive power injection	norm 1	34.49	28.2955	28.3055	28.2974
	norm 2	4.7093	4.5320	4.5348	4.5344
	norm inf.	2.5987	2.6979	2.7009	2.7006
Voltage	norm 1	0.9968	0.9736	0.9738	0.9729
	norm 2	0.103	0.1007	0.1008	0.1007
	norm inf.	0.0276	0.0306	0.0307	0.0307

Table C.14 Residual values for set 4 (p. u.) – Level 3

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	4.4695	4.2564	4.2917	4.2917
	norm 2	0.7196	0.7061	0.7177	0.7176
	norm inf.	0.3683	0.3553	0.3668	0.3668
Reactive power injection	norm 1	32.1291	25.9496	25.9305	25.9304
	norm 2	4.777	4.4794	4.4736	4.4725
	norm inf.	2.8435	2.6206	2.6210	2.6195
Voltage	norm 1	0.9777	0.8904	0.8872	0.8882
	norm 2	0.1018	0.0947	0.0945	0.0946
	norm inf.	0.027	0.0304	0.0304	0.0304

Table C.15 Residual values for set 5 (p. u.) – Level 3

Residual type	**Norm**	**No PMU**	***Method - I***	***Method - II***	***Method - III***
Active power injection	norm 1	4.2148	**4.8973**	**4.9001**	**4.8960**
	norm 2	0.745	**0.7831**	**0.7828**	**0.7821**
	norm inf.	0.3681	**0.3096**	**0.3094**	**0.3092**
Reactive power injection	norm 1	33.5634	27.6232	27.4549	27.3303
	norm 2	4.4505	3.9932	3.9726	3.9536
	norm inf.	1.7777	1.4078	1.3491	1.3166
Voltage	norm 1	0.9817	0.8992	0.8921	0.8934
	norm 2	0.1055	0.0989	0.0987	0.0987
	norm inf.	0.0402	0.0302	0.0310	0.0307

Note: Values in bold refers to no improvement in residuals